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A level-set based multi-material topology optimization method using a reaction diffusion equation

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ABSTRACT

A level-set based multi-material topology optimization method using a reaction diffusion equation is proposed in this paper. Each phase is represented by a combined formulation of different level set functions. This description model is modified from Multi-Material Level Set (MM-LS) topology description model. With a total number of M level set functions, this approach provides a representation of M materials and one void phase (totally $M + 1$ phases). By this approach, the mathematic model of the multi-material topology optimization problem using a reaction diffusion equation is established. With this model, the geometrical complexity of optimal solutions can be easily controlled by appropriately setting a regularization parameter. Some implementation details for solving this model are also presented in this paper. Finally, several typical numerical examples are shown to confirm the effectiveness of the proposed method.

1. Introduction

Topology optimization is often employed to determine the distribution of material in a fixed design domain such that an objective function is minimized under certain constraints at the early design stage. It has been widely studied during the past decades. Several typical methods have already been constructed [1]. Among these methods, the following two have attracted more attention. The homogenization method and its variant, the Solid Isotropic Microstructure with Penalization (SIMP) method are two of the most popular approaches [2,3]. The SIMP method attains great success for its efficient implementation, simple concept and less dependence on predefined parameters. Level set methods for structural topology optimization [4–6] define the structural boundary by level-set function, and they create smooth boundaries. It should be noted that most level set methods for structural topology optimization do not involve mesh-dependent problems which are often encountered in density-based methods.

Most of the research papers are confined to single-material problems. In fact it may be difficult to acquire the best structure by using a single material. For instance, in a problem taking minimum stress as the target function, we can use strong material in domain where the value of the stress is high and weak material in other domain. In recent years there are growing interests in multi-material topology optimization problems, as will be briefly reviewed in the following.

There are a few methods focused on multi-material topology optimization problems. Based on the material distribution concept, Bendsøe and Sigmund [7] proposed a mixture rule of multi-material model in the SIMP method. This model has been used to design multi-physics compliant mechanisms [8]. There are other multi-material models based on density-based methods [9]. In the design of smart structures, various multi-material models were developed to seek the best distribution of the piezoelectric and elastic materials [10–12]. Stegmann and Lund [13] presented two multi-material interpolation schemes as direct generalizations of the well-known SIMP and RAMP material interpolation schemes originally developed for isotropic mixtures of two isotropic material phases. Based on the phase field method and the algorithm combining the binary phase, Tavakoli and Mohseni [14] proposed an alternating active-phase algorithm for multi-material topology optimization problems. Zhou and Wang [15] proposed a general method to solve multiphase structural topology optimization problems, where multi-material phase-field approach based on Cahn–Hilliard equation was employed. Based on the penalization of the objective functional by the multiphase volume constrained Ginzburg–Landau energy functional and the update procedure utilizing the gradient flow of the objective functional by a fractional step projected steepest descent method, Tavakoli [16] introduced a new algorithm to solve multi-material topology optimization problems.

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Nomenclature

ϕ^k	The kth level set function
D	Design domain
Ω^k	The kth material region
Γ^k	The boundary of the kth material
$\mathbf{x}$	A point located in D
M	The number of the level set functions
F	Object functional
G	Constraint functional concerning the volume constraint
$V_{\max}$	Upper limit of the volume constraint
χ_Ω	Characteristic function
λ^k	Lagrange multiplier
$\bar{F}^k$	The Lagrangian
t	The fictitious time
τ	Regularization parameter
φ^k	The kth material
$\mathbf{u}$	Displacement field
$\mathbf{v}$	Virtual displacement
$\boldsymbol{\varepsilon}(\mathbf{u})$	The strain
$\mathbf{D}$	Material elasticity tensor
N	The total number of level set nodes
$\mathbf{f}$	Traction force
$\tilde{u}^0_{i,j}$, $u^0_{k,l}$	Values refer to displacements of the structure
δ_{ij}	Kronecker's delta functions
ν	Poisson's ratio
E^k	Young's modulus of the kth material
$E_{\min}$	Young's modulus in void domain

As previously mentioned, level set methods have shown their merits for structural topology optimization. So there is a great potential to apply level set methods in solving multi-material topology optimization problems. Wang and Wang [17] first proposed a 'color'-level set method for the compliance minimization problem involving multiple materials. The method was further applied to the design of multi-material compliant mechanisms [18] and stress-related optimization problem [19]. Incorporating the piecewise constant level set model into topology optimization, a multiphase level set method of piecewise constants for shape and topology optimization of multi-material piezoelectric actuators with in-plane motion was presented [20]. In addition, Wang and Wang proposed a level-set based variational approach for the design of this class of heterogeneous objects [21]. G. Allaire et al. have enhanced the treatment of material interfaces, a key feature introduced in multi-phase models that is nevertheless typically ignored [22] and Vermaak et al. studied the effects of including material interface properties in the optimization of multi-phase elastic and thermoelastic structures [23].

There are several challenges in solving multiphase structural topology optimization problems as compared to single-material problems. One of the most important challenges is how to set an appropriate topology description model that is able to effectively indicate each distinct phase inside the design domain and avoid overlaps between different phases. Recently Y. Wang et al. proposed a new Multi-Material Level Set (MM-LS) topology description model for topology and shape optimization of structures involving multiple materials [24]. This model has the following advantages: (1) it can effectively indicate each distinct phase; (2) it has an explicit mathematical formulation, which facilitates the sensitivity analysis; (3) it naturally avoids overlaps between each two phases, and guarantees that the design domain contains no redundant phase.

A level-set based topology optimization method using a reaction diffusion equation was proposed by Yamada et al. [25,26]. The novel aspect of this method is the incorporation of level-set based boundary expressions and fictitious interface energy in the topology optimization problem, and the replacement of the original topology optimization problem with a procedure to solve a reaction diffusion equation. It is found that optimal solutions obtained by this method show minimal dependency upon the initial configurations and almost no mesh-dependent problems.

Generally, there are two popular approaches for solving level-set based topology optimization problems [6]. One is the conventional method in which optimization problem is solved by updating the Hamilton–Jacobi partial differential equation (PDE) using shape sensitivities. Another is the method in which parameterization techniques are employed to convert the Hamilton–Jacobi PDE into a simpler set of ordinary differential equations (ODEs). In this work, a multi-material topology optimization method based on the level-set based topology optimization method using a reaction diffusion equation is proposed, which is different from the two above-mentioned approaches. The description model of multi-material topology optimization problems presented here is also different, which is modified from the MM-LS topology description model. In this model, each material should be represented by a combined formulation of all level set functions. The introduced reaction diffusion term allows topological changes that generate new boundaries during the optimization procedure, and re-initialization of the level set function is not required. Furthermore, the geometrical complexity of optimal solutions can be easily controlled.

The rest of this paper is arranged as follows. The model for multi-material topology optimization problems is described in Section 2. Several illustrative examples are given in Section 3. Conclusions are eventually provided in Section 4.

2. The model for multi-material topology optimization problems

2.1. The level-set based topology optimization method using a reaction diffusion equation

The level set function is originally developed by Osher and Sethian [27] with the fundamental goal of tracking the motion of curves and surfaces. In this work, the structural boundary is implicitly represented by the zero level sets. For the multi-material structures, multiple level set functions $\phi^k, k = 1, 2, \ldots, M$ are employed to denote different phases. These level set functions are utilized to define the following subdomains:

$$\begin{cases} \phi^k(\mathbf{x}) > 0 & \forall \mathbf{x} \in \Omega^k \setminus \Gamma^k \\ \phi^k(\mathbf{x}) = 0 & \forall \mathbf{x} \in \Gamma^k, \qquad k = 1, 2, \ldots, M \\ \phi^k(\mathbf{x}) < 0 & \forall \mathbf{x} \in D \setminus \Omega^k \end{cases} \tag{1}$$

where D represents the design domain including all admissible shapes. Ω^k denotes the kth material region with positive value of the kth level set function, Γ^k is the boundary of the kth material, negative value of the kth level set function signifies the domain not containing the kth material. $\mathbf{x}$ represents a point located in D. M is the number of the level set functions. An example for the design domain containing three level set functions is illustrated in Fig. 1.

Firstly, a structural optimization problem that determines the optimal configuration of a domain filled with a solid material is considered. Using a material domain Ω, a void domain, and boundaries Γ, the optimization problems to minimize functional F under a constraint functional G concerning the volume constraint can be formulated as follows:

$$\inf_{\Omega} F(\Omega) = \int_{\Omega} f_d(\mathbf{x})d\Omega + \int_{\Gamma} f_b(\mathbf{x})d\Gamma \tag{2}$$