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Efficient simulation and rendering of realistic motion of one-dimensional flexible objects

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ABSTRACT

In gross motion of flexible one-dimensional (1D) objects such as cables, ropes, chains, ribbons and hair, the assumption of constant length is realistic and reasonable. The motion of the object also appears more natural if the motion or disturbance given at one end attenuates along the length of the object. In an earlier work, variational calculus was used to derive natural and length-preserving transformation of planar and spatial curves and implemented for flexible 1D objects discretized with a large number of straight segments. This paper proposes a novel idea to reduce computational effort and enable real-time and realistic simulation of the motion of flexible 1D objects. The key idea is to represent the flexible 1D object as a spline and move the underlying control polygon with much smaller number of segments. To preserve the length of the curve to within a prescribed tolerance as the control polygon is moved, the control polygon is adaptively modified by subdivision and merging. New theoretical results relating the length of the curve and the angle between the adjacent segments of the control polygon are derived for quadratic and cubic splines. Depending on the prescribed tolerance on length error, the theoretical results are used to obtain threshold angles for subdivision and merging. Simulation results for arbitrarily chosen planar and spatial curves whose one end is subjected to generic input motions are provided to illustrate the approach.

1. Introduction

Motion simulation and rendering of one-dimensional (1D) flexible objects and development of algorithms for real time and realistic or natural motion have been active areas of research in the geometric modeling, CAD and robotics community. The original motivation was the requirement of realism in simulation and display of the motion of cables, ropes, chains, hair, snakes etc. in computer graphics and animation industry. In one of the earlier works, Barzel [1] uses mode shapes of a constrained string model to fake the dynamics of a string. Hergenröether and Däehne [2] discretize the flexible 1D object into a large number of small (linear) rigid objects, each endowed with mass and connected by different kinds of springs and dampers. With appropriate choice of parameter values of mass, spring and damping constant, a physics-based realistic simulation was obtained. Taskıran and Güdükbay [3] have also employed spring–mass system for simulating hair dynamics. In an extension to these works, Güdükbay et al. [4] propose spring mass systems for simulation of elastically deformable models. The main issue in these techniques is to choose or obtain appropriate spring and damping constants for which Natsupakpong and Çavuşoğlu [5] have devised an algorithm for estimating these based on error minimization between FEM and lumped element models. However, the algorithm is not feasible for real-time implementation. Moll and Kavaraki [6] present path planning for flexible 1D objects using minimal energy curves and probabilistic root maps. Ward et al. [7] survey the field of hair modeling, approaching the majority of the existing methods. Grégoire and Schömer [8] and, Spillmann and Tescner [9] use Cosserat model for rod-like solids to model bending and torsion for real-time realistic simulation of flexible parts. Lenoir et al. [10] use Lagrangian formulation posing lumped masses on the control polygon vertices and springs along the curve (resisting bending and stretching) combined with spline refinement techniques, namely sub-division/merging to simulate the motion of flexible objects. In an extension to this work, Theetten et al. [11] generate geometrically exact expressions for deformations of flexible 1D objects by concurrently using beams and spline theory. Goldenthal et al. [12] use a constrained Lagrangian mechanics based approach to handle inextensible cloth simulation—the

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length constraint (in-extensibility) is explicitly enforced on the cloth mesh thereby increasing computational effort especially when the resolution in the cloth model is increased. To overcome the stiff nature of the differential equation in cloth simulation (due to the high compliance along cloth surface normal vector as compared to almost zero compliance for the in-plane extension), Baraff and Witkin [13] use an implicit solver to simulate cloth motion. This is however an iterative procedure and convergence is an issue. Wang et al. [14] use strain limiting algorithms to overcome the stiffness issue. In another work, Mikchevitch et al. [15] use free-form surfaces and flexible beams to model a real time simulator for assembly–disassembly operations. The review paper by Nealen et al. [16] discuss the existing physically deformable models in computer graphics in detail and gives a very good overview. In all the above mentioned works, dynamics is incorporated and a large amount of effort is towards speeding up the computation or improving the accuracy by adjusting the algorithms. However, all of these methods suffer from one or more issues like stability, convergence, computational errors due to mathematical stiffness of the system, dependence on many arbitrary parameters, phantom forces from high residuals, excessive damping/numerical losses or lack of feasibility for real-time implementation.

In contrast to the above mentioned approaches, many authors have focused on viewing this problem from a pure kinematics perspective so that the issues of stability, convergence and choice of parameters do not arise. Brown et al. [17] have presented tying of knots in a rope with a geometric approach where the flexible 1D object is discretized into linear segments connected by joints and the motion of a trailing segment uses a *follow the leader* based strategy. Su et al. [18] use inverse kinematics and energy minimization to approximately preserve length of deformed polyline and a 4-point subdivision scheme is used to obtain smooth $\mathcal{C}^1$ curve from the deformed polyline. In another work, Sreenivasan et al. [19] use the closed-form equations of the classical *tractrix* curve to iteratively compute the motion of all trailing linear segments. They show that the tractrix based approach has the property of attenuating the motion of the segments from the input end, and this results in a more *natural* motion of the flexible 1D object. In a subsequent work, Menon et al. [20] have shown that the tractrix based solution can be derived from a constrained optimization problem involving minimizing the velocity of points on a curve subject to preservation of the length of the curve. In all of these works, only the kinematics of the 1D object is used to impart realism in the simulation and rendering and since the flexible 1D object is discretized into linear rigid segments, the length is explicitly and always preserved.

In the robotics community, motion planning and simulation of *snake* and other robots with large number of rigid links connected by actuated joints have been a continuing research area (see, for example, [21–24] and the references therein). In a robot, if the number of actuated joints is more than six for motion in 3D space and more than three for motion in a plane, then there exist many joint angle sets (configurations) which will achieve the same position and orientation of the end-effector of the robot. The main approach in such *redundant robots* is to effectively use the extra actuated joints for selecting poses/paths which optimize a useful functional—this is called the *resolution of redundancy*. One of the earliest techniques used for resolution of redundancy involved the use of the manipulator Jacobian matrix to minimize joint rotation, velocity, torque or to avoid obstacles and singularities in the path of the robot [21]. This approach involves obtaining the pseudo-inverse of the manipulator Jacobian matrix and can have a complexity of $\mathcal{O}(n^3)$ where n is the number of joint variables. Pseudo-inverse based methods are thus not suitable for motion planning when the numbers of links and joints are large. A second approach developed by Chirikjian and Burdick [22] involves the use of a backbone curve to approximate the redundant robot and the motion planning is done on the backbone curve. The complexity of their algorithm is $\mathcal{O}(n)$ but in this approach the length of the curve may not be preserved. In another approach by Reznik and Lumelsky [23], the motion planning is done in the task space (instead of in the joint space) using the classical tractrix curve. As mentioned earlier, the use of the tractrix curve results in a more natural motion of the robot. The tractrix based algorithm has a complexity of $\mathcal{O}(n)$ where n is the number of rigid links.

In recent past, there has been an increased interest in real-time simulation and rendering of the motion of 1D and 2D flexible objects. This is driven by the need to build simulators for laparoscopy, endoscopy and in the general area of training of medical practitioners where motion of blood vessels, tendons etc., motion inside the gastro-intestinal tract or intestine and actions such as tying of knots and suturing needs to be simulated with a high degree of realism [25,26]. This work has been motivated by the need for developing more realistic simulators for endoscopy and laparoscopic surgeries and is restricted to real-time, efficient and realistic simulation and rendering of the motion of flexible 1D objects.

In this work, the flexible 1D object to be simulated is modeled as a B-spline curve as opposed to being discretized into large number of straight segments. All manipulations are done on the segments of the control polygon which generates the B-spline curve. The main contributions are (a) obtaining new analytic expressions of change in length of a B-spline curve from an initial configuration as the angle between two adjacent segments of its control polygon is changed, (b) use of a tractrix based algorithm on the control polygon of a B-spline curve representing the flexible 1D object, and (c) the development of an adaptive algorithm to approximately preserve the length of the flexible 1D object. The tractrix based algorithm results in a more natural and realistic motion of the curve modeling the flexible 1D object. As the control polygon is moved, the resulting length of the curve is not preserved and the adaptive algorithm is used to sub-divide and merge sides of the control polygon so that a prescribed error tolerance on the length of the curve is maintained at all times during the motion of the curve. Since the angle between the adjacent segments is related to the length of the B-spline curve, monitoring the angle is enough to decide on the subdivision and merging.

Note that the tractrix based algorithm has a complexity of $\mathcal{O}(n)$ where n is the number of segments used to represent the flexible 1D object. However, since the number of sides of the underlying control polygon is much less than n, the complexity of the algorithm can be termed as $\mathcal{O}(1)$. The algorithms for natural and realistic motion planning, the adaptive altering of the control polygon and the mathematical results are illustrated using numerical examples where an arbitrary curve is moved along a generic direction with a prescribed length error tolerance. The efficiency of the developed algorithms are also demonstrated with the numerical examples.

The paper is organized as follows: in Section 2 we briefly present the tractrix based motion of a flexible 1D object. New analytic expressions and results for the length of a quadratic B-spline and cubic B-spline curve in terms of the angle between two adjacent segments of the control polygon are presented. The notion of moving the generating control polygon and resulting change in the length of the spline due to motion of the control polygon is also presented. In Section 3, we present an algorithm to adaptively subdivide and merge edges of a control polygon to maintain the length of a curve to within a specified length error. In Section 4, we present numerical results illustrating our approach for efficient and realistic motion simulation and visualization of motion of flexible 1D objects. In Section 5 we present the conclusions of this work.