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Statistical geometric computation on tolerances for dimensioning[☆]

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HIGHLIGHTS

- A generalized RSS method is proposed for modeling geometric representations of tolerances in the statistical way.
- A set of basic operations over the new tolerance model are proposed to enable tolerance compositing and cascading.
- A set of examples demonstrate applications of the new model in tolerance estimation.
- A tolerance allocation framework based on optimization is also proposed by utilizing analytical forms of the new model.

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ABSTRACT

Dimensions are used to specify the distances between different features in geometric models. These dimensions will often be expressed as a range of allowable dimensions. When considering a group of toleranced dimensions, these ranges can be analyzed as either a worst-case bound on allowable ranges, or as a statistical measure of expected distribution. This paper presents a new geometric model for representing statistically-based tolerance regions. Methods for tolerance estimation and allocation on a geometric model are provided by generalizing root sum square (RSS) methods for compositing and cascading over tolerance zones. This gives us a geometric interpretation of a statistical analysis. Tolerance regions are determined by probabilities of variations of dimensions falling into the region. A dependency graph over dimensions can be represented by a topological graph on which the tolerance cascading and tolerance allocation can be processed. To illustrate applications of this geometric method, we provide examples of tolerance estimation and tolerance allocation on our model. The estimation examples utilize the compositing and cascading operations provided in the analysis method. The allocation examples present an automatic tolerance allocation procedure on the tolerance model. As opposed to existing methods, our allocation method allows us to specify not only a numerical objective of the optimization, but also a statistically-based objective for the geometric shape of the tolerance.

1. Introduction

The quality of parts manufacturing is determined by both design and manufacturing tolerances, which affect the geometric and functional features of finished parts [1]. Engineers are usually required to select appropriate models to describe tolerances so that, on one side, tolerances can be estimated with respect to the design diagram, and on the other side, tolerances can be allocated to each part of the design diagram to meet the target tolerance requirement. Traditionally, there are two major types of tolerance modeling: worst case estimation and statistical variation estimation [2,3]. The statistical variation model relaxes tolerances by considering distributions of variations so that each tolerance is associated with a distribution. Tolerance propagation can be achieved by summing up related tolerances with statistical rules. A statistical analysis model estimates a distribution describing the target tolerance rather than computing extreme values described in the worst case estimation. This paper focus on presenting a new framework for modeling tolerances by statistical variations as well as operations that perform tolerance compositing and cascading on this new model. Since our work is on modeling tolerances, we demonstrate that our model is not only appropriate for tolerance estimation [4] but also for tolerance allocation [5].

1.1. Prior work

Tolerance analysis covers techniques that compute the variations of tolerances for worst case estimation or for statistical

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variation estimation. General reviews on this topic are available in [6,4,7]. A key part of tolerance modeling is representing the tolerance zone in an appropriate way so that variations of tolerances can be obtained and propagated. Worst case estimation models the uncertainties of tolerances by simple geometric entities that are guaranteed to bound variations of dimensions [8,9]. Geometric objects employed are usually higher dimensional polytopes or dual-cones, which represent the allowed region as intervals of coefficients in their algebraic parameterization [10–12]. Part features, such as form, orientation, and size of tolerances can be obtained from such models relatively easily, though further computations on tolerances can still be very complicated. Statistical models consider distributions of variations of tolerances so that each tolerance is associated with a statistical distribution model [13–15,3,16]. Computations on the statistical model, such as tolerance compositing, cascading, or allocation, are far more complicated than those of the worst case estimation.

Several prior methods have proposed addressing tolerance compositing and cascading. Tolerance compositing on statistical models focuses on describing distributions of tolerances as well as providing geometric interpretations (tolerance zones) on models. Tolerance cascading on statistical models studies methods for propagating tolerances along the dimensioning chain or the tolerance dependency graph. Tolerance charting methods are the traditional method for estimating tolerance propagation using engineers' experience. An alternative is computer-aided tolerance charting [17], which aims to reduce the number of iterations of physical trial-and-error runs. However, this method cannot handle complex high dimensional tolerance propagation nor geometric tolerances. Many methods for modeling more general tolerances in both the worst case approach and the statistical variation approach have been proposed [18–20,13,21–26]. One particular approach has used Small Displacements Torsor (SDT) [27,28] to model tolerances [18]. Another approach has used Technologically and Topologically Related Surfaces (TTRS) to form any part as a tree representing the succession of surface associations [24,19,21,23]. Statistically, the cascading of tolerances of parts could be simulated by a Monte Carlo method [20,13].

Several prior methods have also proposed addressing tolerance allocation. Tolerance allocation is an inverse operation to tolerance estimation, in which tolerances are allocated to dimensions with respect to target tolerances as well as design requirements. Chase et al. [5] present an allocation method based on the root square method (RSS). Choi et al. [29] also present an optimal tolerance allocation method on a statistical model. Another allocation method based on optimization is proposed by Forouraghi [30] using a special multi-objective particle swarm optimizer. Loof et al. [31] present an allocation method for linear dimensions by formulating an analytical cost function and analytical constraints to minimize the manufacturing cost, which is hard to extend to handle geometric tolerances. Singh et al. [32] propose an allocation method built on the T-Map model. However, pure geometric operations are hard to implement robustly and to associate directly with probability distribution functions for statistical tolerance analysis.

Recently, some new methods for modeling tolerances and their applications have raised research interest. A representative work is presented by Sahani et al. [33] that describes a systematic solution for the tolerance stack up problem involving geometric characteristics on both the worst case model and the statistical model. Gayton et al. [34] present a method for predicting the defect probability for all allowable production batches on the statistical tolerance model. Beaucaire et al. [14] aim to evaluate a predicted quality for the designer by using the statistical tolerance analysis. Tsai et al. [16] study non-normal distributions and presented a method to analyze the resultant tolerance specification. Barkallah et al. [13] apply statistical analysis methods to study how manufacturing tolerances should be determined for milling operations. Qureshi et al. [15] propose a mathematical formulation of the tolerance analysis integrating quantifier so that a single description of the geometrical requirement can be obtained.

1.2. Our work

Traditional methods usually use Monte Carlo simulation to apply statistical rules to tolerance estimations. Though the method implementation is easy, the simulation time cost is usually very high, which limits the usage over complicated dimensioning diagrams. In addition, as the inverse of tolerance estimation, it is hard to design tolerance allocation methods based on simulations. In this paper, we present a new geometric model for statistical tolerances that avoids simulations. Our goal is to decompose the complicated analytical computation of the conventional statistical tolerance analysis into a series of simple geometric computations, without losing the intrinsic statistical meaning. We first use a multivariate statistical model to represent or approximate the tolerance distribution for one single part in the system, which is associated with several dimensions that describe part features. For a given confidence value, we can determine a geometric region that describes the tolerance zone. Those regions are usually bounded by high dimension ellipsoids. We propose a set of operations on the statistical model so that geometric zones from different parts can be cascaded and tolerances can be propagated along the tolerance dependency graph. We also propose an operation to extract the lower degree of freedom (DOF) information from a high DOF representation so as to facilitate the tolerance estimation, such as the clearance estimation of an assembly graph. Since we propose a new model for statistical tolerance computation, we demonstrate its applications both on tolerance estimation and on tolerance allocation. We argue that our new model contributes to the computation of tolerances in both statistical and geometric ways.

To summarize contributions of this paper:

- We generalized the RSS method from the space of one DOF to the space of high DOFs and proposed complete geometric representations of tolerances without sacrificing the underlying statistical meaning.
- On our tolerance model, we described a set of operations as a part of the generalized RSS method to enable tolerance compositing and cascading. Those operations have both geometric and statistical explanations.
- We proposed applications for tolerance estimation with our tolerance model. The estimation utilizes operations on tolerance zones so that tolerances can be composited and propagated.
- We also proposed applications for tolerance allocation with our tolerance model, that utilize the analytical representation of our model and provide an automatic allocation paradigm based on optimizing cost. As opposed to conventional methods, our allocation method allows us to specify not only numerical constraints, such as zone areas, but also geometric constraints, such as zone principal directions.

This paper is organized as follows. Section 2 explains tolerance compositing. We build tolerance zones for primitives within this section. Section 3 explains the generalized RSS method and a set of operations based on the statistical model. Section 4 proposes applications on our tolerance model, including tolerance estimation and tolerance allocation.

2. Tolerance compositing

In this section, we use the multivariate normal distribution (MND) to approximate distributions of tolerances. We also use the χ^2 distribution to yield geometric representations of tolerances.