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Direct diffeomorphic reparameterization for correspondence optimization in statistical shape modeling[☆]

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HIGHLIGHTS

- We propose an approach for optimizing shape correspondence across a population.
- B-splines are used for shape representation and reparameterization.
- The quality measure of the statistical shape model is the description length.
- An adjoint method for deriving analytical sensitivity is developed.
- The approach improves shape correspondence in a group-wise manner.

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ABSTRACT

In this paper, we propose an efficient optimization approach for obtaining shape correspondence across a group of objects for statistical shape modeling. With each shape represented in a B-spline based parametric form, the correspondence across the shape population is cast as an issue of seeking a reparameterization for each shape so that a quality measure of the resulting shape correspondence across the group is optimized. The quality measure is the description length of the covariance matrix of the shape population, with landmarks sampled on each shape. The movement of landmarks on each B-spline shape is controlled by the reparameterization of the B-spline shape. The reparameterization itself is also represented with B-splines and B-spline coefficients are used as optimization parameters. We have developed formulations for ensuring the bijectivity of the reparameterization. A gradient-based optimization approach is developed, including techniques such as constraint aggregation and adjoint sensitivity for efficient, direct diffeomorphic reparameterization of landmarks to improve the group-wise shape correspondence. Numerical experiments on both synthetic and real 2D and 3D data sets demonstrate the efficiency and effectiveness of the proposed approach.

1. Introduction

A *statistical shape model* (SSM) provides a compact characterization of the shape variability in a set of shapes. It was initially used as a tool for facilitating automatic image segmentation [1,2]. It has since seen many other applications including facial recognition [3], computer animation [4], medical diagnosis [5,6], patient-specific modeling [7–10] and human body modeling [11], to name but a few. Finding correspondence across all shape instances is a fundamental task in building SSM. Manual identification of landmarks is effective under some circumstances but in general is not a reliable strategy since it tends to be subjective, time-consuming, error prone, and difficult to be applied in large scale data sets [12]. Consequently, methods for automatically identifying the shape correspondence have been a major research focus in the field.

The automatic identification of the shape correspondence across a set of objects can be achieved by manipulating correspondence either in the object space or in the parameter space. Thus far, one of the most common approaches to achieving shape correspondence is through deforming in the object space a template shape to each shape instance in the training set, and the found pairwise correspondences are then propagated through the common template reference to form the group-wise correspondence. Group-wise registration has also been developed [13]. In such a deformation based correspondence manipulation approach, the

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deformed landmarks may not actually lie on the object shape before the optimization convergence is achieved. Further, the deformation usually reduces to a problem where a "similarity" measure between the template and each shape instance is minimized and some regularization constraints are satisfied. Typically such a measure is related to geometric descriptors such as spatial distance and shape feature, and the optimization is essentially a rigid or non-rigid registration problem [14–17]. However, these geometric descriptors and the regularization constraints are not necessarily a good basis for correspondence.

An alternative is to manipulate the correspondence through reparameterizing the shapes in the parameter space. For example, in [18], objects of spherical topology are mapped to a sphere and correspondence is manipulated through concatenations of symmetric theta transformations on the spherical map. Reparameterization of shapes in the parameter space thus allows convenient manipulation of correspondence of surface points by simply changing point parameters. Reparameterized points always lie on the object shape during the optimization process. As such, the reparameterization lends itself to a more principled approach for establishing correspondence: optimizing the quality of the resulting statistical models. During the past few years, SSM quality measures have evolved from the model covariance trace [19], to the model covariance determinant [20], and finally to the *Description Length* (DL) [18,21,22] and its simplification [23] or variants [24]. This information theoretic objective function of description length has shown to be an effective measure [21] for the population-based correspondence optimization.

Although the population-based approach to shape correspondence does not require the pre-selection of a template and tends to provide a more faithful characterization of the variability pattern, this approach is still far from being widely used to build SSMs due to its low efficiency in identifying optimal correspondence across the shape population. In the minimum description length based group-wise correspondence optimization approach originated in [21], the group-wise shape correspondence search consists of successive small-scale optimizations, each of which uses only a few optimization variables to relocate landmarks in a local region of each shape instance. In each optimization, only landmarks in local regions are moved. This necessitates a huge number of successive optimizations to manipulate all the landmarks, thus leading to inefficiency. Some researchers use analytical gradient formula whenever possible to speed up the gradient evaluation [25,26]. However, in these implementations, the landmark positions in the training set shapes are non-differentiable with respect to optimization variables, the gradients are thus only partially analytical. In [27], spline representation of 2D shapes is proposed so a full analytical gradient of reparameterization can be derived.

In our proposed approach, we cast shape correspondence as an issue of seeking optimal reparameterization $\mathbf{D}(\mathbf{u})$ of the parametric field $\mathbf{u}$ of each shape so that a quality measure f of the resulting shape correspondence across a group of objects is optimized. The reparameterization is applied to the parametric domain of parameterized curves or surfaces. Our SSM is based on the point-distribution model [28]. In our approach, each landmark point $\mathbf{S}(\mathbf{u})$ in a given shape is changed to $\mathbf{S}(\mathbf{D}(\mathbf{u}))$ in order to improve correspondence via the reparameterization $\mathbf{D}(\mathbf{u})$. Our approach thus requires the parameterization of each shape, that is, every point $\mathbf{x}$ of the shape in the physical space is mapped to a point $\mathbf{u}$ in the parametric domain. In our implementation, we choose the B-spline representation $\boldsymbol{S}(\mathbf{u})$ of each shape instance, which can be reconstructed from triangular mesh representation of 3D objects. The parametric domain then undergoes a reparameterization represented via another tensor-product B-spline $\mathbf{D}(\mathbf{u})$ with B-spline coefficients $\mathbf{b}$ as the optimization parameters. We choose the description length as the objective function of the shape correspondence.

Fig. 1 illustrates the proposed idea. A group of hand contours are shown in Fig. 1(a). Each shape is represented with B-splines, and Fig. 1(b) shows such a B-spline representation for one shape with control points and knots. Initially landmarks are uniformly sampled over the parameter domain of the B-spline shape $\mathbf{S}(u)$ as shown in Fig. 1(c). To change the landmark positions, reparameterization $\mathbf{D}(u)$ is applied to the parameter domain of each B-spline shape. This reparameterization is also represented with B-splines as shown in Fig. 1(d) where each red point represents a B-spline coefficient for the reparameterization. The landmarks are redistributed as shown in Fig. 1(e) after the reparameterization. The landmark redistribution can be seen from the four highlighted landmarks, where a, b, c, d moved to A, B, C, D respectively over the other side of the finger tips.

The salient feature of this approach is as follows:

- *Diffeomorphic through B-splines*. Existing technique [18] for reparameterization concatenates a series of simple homeomorphic mappings. One optimization run with this reparameterization technique leads to the deformation of a local parametric region and it cannot provide any information on the search direction for subsequent local deformations in other regions. Therefore it requires the concatenation of a large number of simple mappings and causes severe inefficiency (see Section 5.1). Instead of concatenation of many local mappings, we propose the use of single B-spline function to directly represent the diffeomorphic reparameterization $\mathbf{D}(\mathbf{u})$ for the parameterization $\mathbf{u}$ of each shape instance $\mathbf{S}(\mathbf{u})$. The injectivity for the reparameterization is guaranteed by enforcing the Jacobian positivity constraint.
- *Full differentiability of the objective function f (i.e. description length) with respect to reparameterization variables* $\mathbf{b}$. The objective function f (i.e. description length) is a function of landmark positions. The landmark positions in each shape are differentiable with respect to reparameterization parameters $\mathbf{b}$ due to the parametric representation $\mathbf{S}(\mathbf{u})$ of each shape and diffeomorphic reparameterization $\mathbf{D}(\mathbf{u})$. This ensures that the description length is fully differentiable with respect to the reparameterization variables $\mathbf{b}$.

The direct diffeomorphic reparameterization based formulation for SSM leads to an optimization problem with a large number of constraints (for enforcing the injectivity of reparameterization) and a large number of optimization variables (i.e. B-spline coefficients for reparameterization). Due to the full differentiability of the objective function f (i.e. description length) with respect to reparameterization variables $\mathbf{b}$, a gradient-based optimization approach can be developed to ensure fast convergence. More specifically, the following optimization techniques are developed.

- *Constraint aggregation*. The B-spline based diffeomorphic reparameterization leads to a large number of constraints on Jacobians for ensuring the mapping is bijective. In order to facilitate fast convergence in gradient-based optimization, a constraint aggregation technique is used where the large number of constraints are aggregated into one or a few constraints.
- *Adjoint method for computing sensitivity*. The adjoint approach is used to compute the sensitivity of the objective function with respect to reparameterization parameters $\mathbf{b}$, which is more efficient than direct differentiation of the objective function f. In computing the sensitivity of the description length w.r.t. optimization variables $\boldsymbol{b}$, eigenvalues of the covariance matrix and their derivatives are needed. Since each eigenanalysis is expensive, the adjoint method is thus especially efficient for computing the sensitivity in this kind of optimization problems that