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Canal surfaces with rational contour curves and blends bypassing the obstacles[☆]

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HIGHLIGHTS

- The rationality of generalized contours on rational canal surfaces is studied.
- The contour method is used for computing PN blends between two canal surfaces.
- The constructed blends can easily satisfy certain constrains, e.g. avoiding obstacles.
- Only one SOS decomposition for all canal surfaces with the same silhouette is needed.

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ABSTRACT

In this paper, we will present an algebraic condition, see (20), which guarantees that a canal surface, given by its rational medial axis transform (MAT), possesses rational generalized contours (i.e., contour curves with respect to a given viewpoint). The remaining computational problem of this approach is how to find the right viewpoint. The canal surfaces fulfilling this distinguished property are suitable for being taken as modeling primitives when some rational approximations of canal surfaces are required. Mainly, we will focus on the low-degree cases such as quadratic and cubic MATs that are especially useful for applications. To document a practical usefulness of the presented approach, we designed and implemented two simple algorithms for computing rational offset blends between two canal surfaces based on the contour method which do not need any further advanced formalism (as e.g. interpolations with MPH curves). A main advantage of the designed blending technique is its simplicity and also an adaptivity to choose a suitable blend satisfying certain constrains (avoiding obstacles, bypassing other objects, etc.). Compared to other similar methods, our approach requires only one SOS decomposition for the whole family of rational canal surfaces sharing the same silhouette, which significantly simplifies the computational complexity.

1. Introduction

Silhouette curves are planar curves enclosing the projection of a spatial object into plane. They not only help to create a first visual perception of the shape of the studied object but they have also many practical applications—for instance, in reconstruction of the 3D object from a 2D image, in rendering, in computing the visible area, when deleting invisible curves. There exist two types of silhouettes—parallel (corresponding to the viewpoints at the infinity) and perspective (corresponding to the real viewpoints at a finite distance). The preimages of the silhouette curves on the studied 3D object are called contour curves. We recall that in some papers the notion silhouette curves is used simultaneously also for the contour curves — however, we will strictly distinguish it in what follows. More details about silhouettes/contours and their computation and related topics can be found e.g. in [1–4].

Silhouette and contour curves are very complex even for simple surfaces and in most cases it is difficult to represent them accurately (and finding their NURBS representation, if it exists, is even more complicated). In this paper, we will focus only on one of the most important classes of technical surfaces, namely on the rational canal surfaces, whose silhouettes were already studied e.g. in [5–7]. We will investigate the contour curves (solely with respect to the parallel projection) from the point of view of their

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rationality. In addition, we will show that they are very useful e.g. for the construction of adaptive blending surfaces between two canal surfaces satisfying some given constraints (for instance avoiding obstacles). This approach is based on the construction of a single rational silhouette curve which is common for a whole family of different rational canal surfaces and then choosing a suitable surface with prescribed properties with help of a certain distance function.

Let us recall that the canal surfaces, see e.g. [8], are defined as envelopes of one parameter families of spheres in 3-space. Nowadays they are very popular in Computer-Aided Design as they can be used for constructing simple blends between two given surfaces, see [9] and references therein for a detailed overview of blending techniques. Among others, all surfaces of revolution belong to canal surfaces. For a constant radius of moving spheres we speak about pipe surfaces. A special (and probably the most investigated) subfamily of the canal surfaces are Dupin cyclides which can be defined as the envelopes of all spheres touching three given spheres, see e.g. [10–13].

It was proved in [14] that any canal surface with a rational spine curve (a set of all centers of moving generating spheres) and a rational radius function has a rational parameterization. This result followed an equivalent result from [15] for pipe surfaces. A technique for computing rational parameterizations of canal surfaces was presented in [16]. Minimal rational parameterizations of canal surfaces were thoroughly studied in [17]. Let us emphasize that although the canal surfaces with rational spine curves and rational radii always possess exact rational parameterizations, approximate parameterization techniques are also investigated in connection with them, see e.g. [18]. This is caused by the computational difficulty of decomposing a rational function into a sum of two squares (SOS) over reals, which is a necessary part of the parameterization algorithm from [16] (and we will see that the SOS decomposition will occur also in the approach presented in this paper when exact rational parameterizations are required).

Recently a 'contour method' for parameterizing (some classes of) canal surfaces has been introduced in [18,19]. This approach is based on the main idea that if the parameterizations of the spine curve and one curve on the canal surface (different from the characteristic circles) are known and they are corresponding in parameter we can easily find a rational parameterization of the canal surface by rotating the points of the given curve along the tangents of the spine curve. In [19], a condition guaranteeing the rationality of the special curves on canal surfaces called contour curves, which were then used for a computation of parameterizations of canal surfaces, was investigated. The presented approach extended the results about rational spatial Minkowski Pythagorean hodograph curves and the associated planar (Euclidean) Pythagorean hodograph curves from [20] and related them with the contour curves of canal surfaces given by their medial axis transforms.

In this paper we will follow the approach from the aforementioned papers and extend the notion of 'contour curves' (originally related to the coordinate planes and coordinate axis directions only) to general planes and general directions, too. This allows us to apply the contour method to a broader class of canal surfaces and hence a main drawback of the original method (dependence on the choice of a suitable coordinate system) is eliminated. We will formulate a condition on the medial axis transform of the canal surface which guarantees the rationality of the contour curves with respect to a given general direction (viewpoint at infinity), and thus also the rationality (and the rational offsets property) of the obtained canal surface parameterizations. In addition, we will mention the existence of real contour curves which, of course, do not have to exist for a chosen direction and a chosen part of the given canal surface, in general.

The remainder of this paper is organized as follows. Section 2 summarizes some fundamental facts concerning the canal surfaces, contour curves and SOS decomposition problem. As a motivation, in Section 3 we recall several results about contour curves on implicitly given canal surfaces. In Section 4, general contour curves are introduced and the condition which must be satisfied by the MAT and that guarantees the rationality of the contour curves is derived and discussed. The main part of the paper is devoted to the low-degree cases such as quadratic and cubic MATs that are especially useful for practical applications. In Section 5, the polynomial quadratic and cubic medial axis transforms are thoroughly studied from the point of view of the rationality of the associated contour curves. We will show how to find a suitable viewpoint which guarantees the rationality of the contour curves on this type of canal surfaces (i.e., with quadratic or cubic MATs). Special investigations will be devoted to pipe surfaces with low-degree MATs. In Section 6 we will focus on the operation of blending with rational contours. A simple direct method (which does not require in most cases another advanced modeling technique as e.g. modeling with MPH curves) for computing rational offset blends between two canal surfaces based on the contour method will be formulated. A main advantage of the designed blending technique is its simplicity and especially its usefulness for constructing blends satisfying certain constrains, e.g. when avoiding obstacles or bypassing other objects is required. Compared to other methods our approach needs only one SOS decomposition for the whole family of rational canal surfaces sharing the same silhouette, see Fig. 1. The designed algorithms are presented in several examples. In Section 7, we conclude the paper.

2. Preliminaries

We start with short recalling some fundamental properties of canal surfaces which will be used in the following sections. We also mention the problem of sum of squares decomposition of non-negative polynomials over reals playing an important role in our method.

2.1. Canal surfaces

A *canal surface* $\mathcal{S}$ is the envelope of a 1-parameter family of spheres F whose centers trace a curve $\mathbf{m}$ in $\mathbb{R}^3$ and possess radii r, i.e.,

$$F(t): \ \|\mathbf{x}-\mathbf{m}(t)\|^2 - r(t)^2 = 0, \quad t \in I, \tag{1}$$

where $\mathbf{x} = (x, y, z)^\top$. The curve $\mathbf{m}$ is called the *spine curve* and r the *radius function* of $\mathcal{S}$. For constant r we obtain a *pipe surface*. The defining equations for the canal surface $\mathcal{S}$ are

$$F(t) = 0, \qquad F'(t) = 0, \tag{2}$$

where F' is the derivative of F with respect to t. When we eliminate the parameter t from (2) one can get the corresponding implicit equation $f(\mathbf{x}) = 0$ of $\mathcal{S}$. The linear equation $F' = 0$ describes the plane with the normal vector $\mathbf{m}'$, i.e., perpendicular to the spine curve $\mathbf{m}$. Thus the canal surface $\mathcal{S}$ contains a one parameter set of the so called *characteristic circles* $F \cap F'$ and canal surfaces belong to the so called *ringed surfaces*. It can be proved that the envelope (i.e., the canal surface) is real iff the condition

$$\|\mathbf{m}'(t)\|^2 - r'^2(t) \geq 0 \tag{3}$$

is fulfilled for all $t \in I$.

By appending the corresponding sphere radii r to the points of the spine curve (or the *skeleton*, or the *medial axis*) we obtain the *medial axis transform* MAT, i.e., the curve $\overline{\mathbf{m}}$. For the sake of clarity, we identify the canal surface $\mathcal{S} \subset \mathbb{R}^3$ with its medial axis transform