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Optimizing conformality of NURBS surfaces by general bilinear transformations[☆]

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HIGHLIGHTS

- We give explicit representations of the general bilinear reparameterized surfaces.
- A scheme is given to construct the general bilinear reparameterized NURBS surfaces.
- An optimization method is given to improve the conformality of NURBS surfaces.

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ABSTRACT

The conformality of NURBS surfaces greatly affects the results of rendering and tessellation applications. To improve the conformality of NURBS surfaces, an optimization algorithm using general bilinear transformations is presented in this paper. The conformality energy is first formulated and its numerical approximation is then constructed using the composite simpson's rule. The initial general bilinear transformation is obtained by approximating the conformal mapping of its 3D discretized mesh using a least square method, which is further optimized by the Levenberg–Marquardt method. Examples are given to show the performance of our algorithm for rendering and tessellation applications.

1. Introduction

Freeform surfaces play an increasingly important role in contemporary Computer Aided Design (CAD). The results of most surface algorithms [1–13] for surface rendering (e.g. texture mapping), tessellation and blending applications are highly dependent on the surface parameterization. A NURBS surface has an intrinsic rational piecewise polynomial mapping from the 3D surface to the 2D parameter domain (see Fig. 1). By surface reparameterizations [14,11–13], the surface may have infinitely many different parameterizations. Depending on where and how it will be used, one may need to find a suitable or optimal parameterization out of the infinitely many, or to convert the given parameterization into another (more) suitable one [14,11–13]. In many applications, such as texture mapping, surface tessellation, surface matching and registration, it is highly desirable that the parameterization is shape preserving (conformal) i.e. maps an elementary circle of the parameter domain to an elementary circle of the surface. At the same time, the surface modifications (changing the surface control points and/or weights) and surface fitting in reverse engineering both may introduce NURBS surfaces with parameterizations far from being conformal. From our point of view, the lack of conformal parameterizations is the bottleneck for NURBS surfaces to achieve high quality results for rendering and tessellation applications. Moreover, a conformal parameterization will lead to more robust and stable computations for derivative based algorithms such as surface intersection, curvature computation, and so on [1,6,9,11].

1.1. Related works

We focus on reviewing on NURBS curve and surface parameterization methods. For triangle meshes and subdivision surfaces, we

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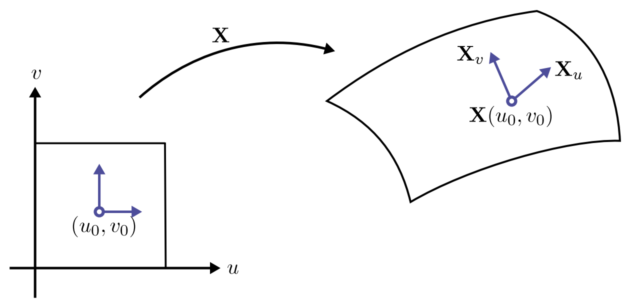

Fig. 1. The intrinsic mapping of a NURBS surface. A 2D point (u_0, v_0) in the parameter domain is mapped to a 3D point $\mathbf{X}(u_0, v_0)$ on the surface. The two partial derivatives $\mathbf{X}_u(u_0, v_0)$ and $\mathbf{X}_v(u_0, v_0)$ at the point $\mathbf{X}(u_0, v_0)$ may be not orthogonal and have different norm lengths.

concentrate on analyzing the difference between their parameterization and freeform surface parameterization, a detailed review of mesh and subdivision surface parameterization techniques being beyond the scope of this article. For the details of mesh parameterization, the reader is recommended to see the survey paper by Floater and Hormann [15] and the references therein. For the details of subdivision surface parameterization, see the papers by He et al. [16,17] for the latest progress.

In the past twenty years, how to achieve optimal parameterization of Bézier curves has been studied extensively in the literatures such as [18–28]. Farouki [19] identified arc-length parameterization as the optimal parameterization of Bézier curves. By minimizing an integral which measures the deviation from arc-length parameterization, the optimal representation is obtained by solving a quadratic equation. Jüttler [20] presented a simplified approach to Farouki's result by using a back substitution in the integral. Costantini et al. [18] obtained closer approximations to the arc-length parameterization by applying composite reparameterizations to Bézier curves. Farin [29] showed that for a circular arc represented by its rational quadratic representation, its parameterization is a chord length one. Motivated by Farin's work, Lu [30] identified a family of curves that can be parameterized by chord length. Yang et al. [31] presented a reparameterization method to approximate the arc-angle parameterization of any planar curve by C^1 piecewise Möbius transformations.

The parameterization of triangle meshes has been studied extensively in the last decade and still remains as a hot topic until now [15,32–35]. The main purpose of the research on the parameterization of triangle meshes is to construct a suitable, bijective mapping between the triangle mesh embedded in 3D and a simple 2D domain, referred to as the parameter space or parameter domain. To minimize the parameterization distortion in either angles or areas, many different algorithms have been proposed in the literature [15,33,35]. Also He et al. presented algorithms for parameterizing subdivision surfaces in [16,17]. As the NURBS surface already has an intrinsic rational polynomial mapping (see Fig. 1) from the 2D parameter domain (a rectangle) to the 3D surface, its parameterization has some specific properties different from the parameterization of triangle meshes and subdivision surfaces. Firstly the NURBS surface has an intrinsic mapping already and we do not need to construct an initial surface mapping from the 3D surface to the 2D parameter domain, which is the case for triangle meshes and subdivision surfaces. Secondly the parameterization of NURBS surface is a continuous rational polynomial mapping while those of triangle meshes and subdivision surfaces are discrete, usually defined by the correspondence between their vertices and the correspondence of points inside the triangles/quads is obtained from the vertices correspondence by interpolation techniques. If we convert the NURBS surface into a triangle mesh and apply the mesh parameterization method, some parameterization results can be obtained subsequently. However, there is one main drawback for this kind of methods. The resultant surface representation is not NURBS anymore, which is problematic for subsequent CAD algorithms designed for freeform surfaces. Though we can reconstruct the NURBS surface by traditional least-square fitting methods from the triangle parameterization, neither the surface shape nor the triangle parameterization are preserved precisely during the fitting procedure, which is not allowed for high accurate CAD applications.

The aim of this paper is to optimize the conformality of given NURBS surfaces for CAD applications such as surface visualization, surface tessellation, surface intersection, curvature computation and so on. Most of all, we want to keep the surface geometry (the surface shape) unchanged and let the resultant surfaces represented as NURBS surfaces, which is preferable and convenient for the algorithms designed for the Computer Aided Design applications mentioned above. The method presented in this paper utilizes the general bilinear transformation, which satisfies both the surface shape and representation requirements.

Compared with the quantitative study of NURBS curve, triangle mesh and subdivision surface parameterization [18,19,15–17,20,21,33,22–28,35], little attention has been paid to the NURBS surface parameterization until now. The results of rendering and tessellation applications for NURBS surfaces largely depend on the parameterization quality. To perform texture mappings on a NURBS surface, the parametric coordinate of the surface is usually reused as the texture coordinate. If the parameterization is far from being conformal, there will be large distortion of the texture image on the surface (see Fig. 2(b)). To tessellate a NURBS surface, most existing algorithms [4,6,7] map a triangulation of the parameter domain onto the surface. Similar to texture mapping, the final tessellation results largely depend on the surface parameterization (see Fig. 2(c)). A conformal surface parameterization not only preserves the appearance of texture, but also avoids degenerate elements for the tessellation application. Moreover, a conformal parameterization will lead to more robust and stable computations for derivative based algorithms such as surface intersection, curvature computation, and so on [1,6,9,11].

Yang et al. [36] presented an algorithm to improve the Bézier surface parameterization based on Möbius transformations [37,36], which can change only the distribution of iso-parametric curves, but not the shape of them. To obtain more uniform iso-parametric curves, a rational bilinear reparameterization algorithm was also presented in [36]. However, only the uniformity of iso-parametric curves was considered. Furthermore, the rational bilinear reparameterization coefficients are determined by a trivial interpolation method, which is only suitable for a special surface case. To obtain more uniform and orthogonal iso-parametric curves for rational Bézier surfaces, Yang et al. [38] presented an optimization algorithm to minimize the nonlinear energy measuring uniformity and orthogonality deviations using the rational bilinear reparameterizations, which produces a better parameterization with the cost of degree elevation. However both the Möbius transformation and rational bilinear transformation presented in [36,38] suffer from the fact that they always map the four surface boundary curves to themselves in the transformation. As a result, the angles between partial derivative vectors at the four corner points remain unchanged. The surface parameterization can only be improved to some extent using the Möbius transformations and rational bilinear transformations.

1.2. Algorithm overview

To remove the boundary constraints of the Möbius transformation and the rational bilinear transformation, a general rational bilinear transformation is presented in this paper. Based on