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A variational-difference numerical method for designing progressive-addition lenses[☆]

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HIGHLIGHTS

- We propose a variational-difference numerical method for designing progressive-addition lenses.
- The method can be very easily understood and implemented by optical engineers.
- The method can provide satisfactory designs for optical engineers in several seconds.
- The method can be a powerful candidate tool for designing various free-form lenses.

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ABSTRACT

We propose a variational-difference method for designing the optical free form surface of progressive-addition lenses (PALs). The PAL, which has a front surface with three important zones including the far-view, near-view and intermediate zones, is often used to remedy presbyopia by distributing optical powers of the three zones progressively and smoothly. The problem for designing PALs could be viewed as a functional minimization problem. Compared with the existing literature which solved the problem by the B-spline finite element method, the essence of the proposed variational-difference numerical method lies in minimizing the functional directly by finite difference method and/or numerical quadratures rather than in approximating the solution of the corresponding Euler–Lagrange equation to the functional. It is very easily understood and implemented by optical engineers, and the numerical results indicate that it can produce satisfactory designs for optical engineers in several seconds. We believe that our method can be a powerful candidate tool for designing various specifications of PALs.

1. Introduction

When we reach about forty years old, the natural process of aging begins to affect our vision, and the lens of our eyes thickens and progressively loses its flexibility to the point where we have trouble focusing on near points. This causes presbyopia. Therefore, it is natural that people with presbyopia need their vision to be corrected by wearing spectacle lenses. People can often use single-vision lenses to correct this problem. However, although these can enable very good vision for nearby regions, people need to take them off in order to have good vision for distance regions. To avoid this inconvenience, more complicated lenses such as the bifocal lens, trifocal lens and progressive-addition lens (PAL) have been designed.

Bifocal lenses, which were first invented by Benjamin Franklin in 1784 because he suffered from poor vision at that time, can be divided into two parts, the top half for viewing at distance and the bottom half for reading; trifocal lenses are made up of three parts, with the addition of a part for viewing at intermediate region. A major drawback for these two kinds of lenses is the vision jump when the eyes move from seeing far-distance to near-distance objects. PALs, also known as no-line bifocal lenses, can remove the vision jump drawback. Fig. 1 shows a schematic illustration of PALs. As shown in Fig. 1(a), a PAL has a front surface with three different view zones, including distance-view (i.e. far-view), near-view and intermediate zones. More precisely, a PAL has a large far-vision area with low refractive optical power in

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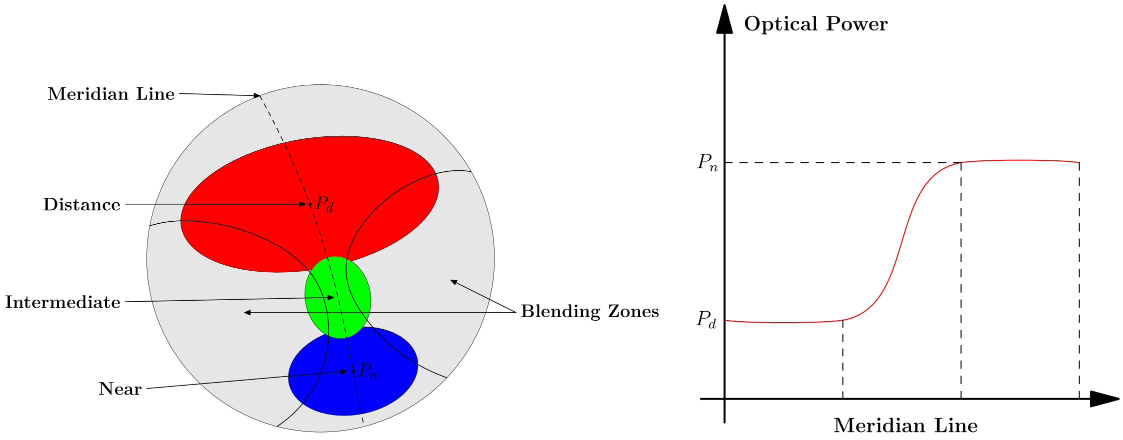

(a) Three different view zones of a PAL.

(b) The distribution of refractive optical power.

Fig. 1. Schematic illustration of a PAL.

the upper part and a small near-vision area with high refractive optical power in the lower part, while an intermediate vision area where the optical power varies progressively and smoothly is used to connect with the upper and lower parts of the PAL. Fig. 1(b) shows a typical distribution of refractive optical power on the three different view zones of a PAL along the meridian line. In general, a good PAL design requires that the optical power on the far-view, near-view and intermediate zones be progressively and smoothly changed according to every patient's prescription, and that the astigmatism on the three zones be as small as possible simultaneously. However, the remaining parts, which are called as blending zones shown in Fig. 1(a), also inevitably have astigmatism because of the prescribed power distribution along the three zones. Although the blending zones are the least frequently used by spectacle wearers, the astigmatism will bring them a bad visual feeling and make them uncomfortable. Therefore, a good PAL design also needs to require that the blending zones have relatively low astigmatism.

As early as 1907, British optometrist Owen Aves made a prototype design for PALs [1,2]. His design idea came from the shape of elephant's trunk, which consisted of a conical back surface and a cylindrical front with opposing axes in order to create a power progression. This design was the prototype of modern PALs, but it was not commercialized at that time. Since the invention of the first modern design and entry into the marketplace around the 1960s, PALs have been gradually accepted by worldwide customers due to their ability to eliminate the vision jump line between the far-view and near-view portions of the lens and offer spectacle wearers a smooth transition between different vision zones. Nowadays, PALs have gained worldwide acceptance as the high-performance spectacle lenses used in the correction of presbyopia and currently account for more than half of all multi-focal lens sales. Although early progressive lens designs have had great success in providing presbyopic patients with more comfortable vision, lens designers seem to be approaching a limiting state [3]. Because the visual requirements of spectacle lens wearers vary greatly from person to person, it has been understood for some time that the traditional "one-size-fits-all" progressive lens design framework is no longer suitable for every progressive lens wearer. Therefore, designers need to tailor more suitable and specialized spectacle lenses by considering the unique visual requirements of the individual progressive lens wearer. Thus, the advances in design methods are becoming ever more significant for designing the next generation PALs.

Designing PALs is often regarded as a very complicated mathematical problem [4–10]. In general, the design methods can be divided into two categories, direct methods and indirect methods. In a direct method, such as the research work of Winthrop [4,5] and Baudart, Ahsbahs and Miege [6], the refractive optical power is first assigned along a line called the meridian line (shown in Fig. 1) on the lens, then the surface on the lens is generated from the meridian line by prescribing curves which are transverse to it. The shapes of these curves are chosen to have the desired surface curvature on the meridian line. However, the performance of such design methods is often less than satisfactory because there is no effective control over the distribution of the astigmatism. For the indirect method, such as the method proposed by Loos et al. [7,8], the lens design can be simplified as an optimization problem or a functional minimization problem. In such a method, a cost function (functional), which attempts to balance between reaching the desired distribution of refractive optical power and the unwanted astigmatism, will be devised and beforehand given. The design objective is to minimize the cost function (functional) by numerical methods. The indirect method is often more effective and powerful, and it can be quickly implemented by powerful computer simulations. It can also provide more precise control for the distribution of the optical power and astigmatism on the lens surface. Therefore, it represents a powerful candidate tool for designing the customized PALs.

In this paper, we focus on the indirect methods for designing PALs. Based on a functional minimization mathematical model, we propose an efficient variational-difference numerical method for solving the problem. Compared with the existing literature which solved the problem by the *B*-spline finite element method [7,9,10], the essence of the proposed variational-difference numerical method lies in minimizing the functional directly by the finite difference method rather than in approximating the solution of the corresponding Euler–Lagrange equation to the functional. It is very easily understood and implemented by optical engineers, and its memory and computational costs are smaller than that of the *B*-spline finite element method. Our numerical results indicate that it could produce satisfactory design results for optical engineers in several seconds.

The rest of the paper is organized as follows. In the next section, we briefly present a mathematical model based on a quadratic functional minimization problem for designing PALs. In Section 3, a variational-difference numerical method is proposed for solving the quadratic functional minimization problem. In Section 4, numerical results are presented to demonstrate the high performances of the proposed numerical method. Finally, Section 5 concludes the paper with a summary and future research plans.