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Review

A comprehensive study of three dimensional tolerance analysis methods

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HIGHLIGHTS

- Introduce four major 3D tolerance analysis models briefly.
- Make a comprehensive comparison and discussion between them.
- Expound the connotation of 3D tolerance analysis.
- Present a perspective overview of the future research about 3D tolerance analysis.

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ABSTRACT

Three dimensional (3D) tolerance analysis is an innovative method which represents and transfers tolerance in 3D space. The advantage of 3D method is taking both dimensional and geometric tolerances into consideration, compared with traditional 1/2D tolerance methods considering dimensional tolerances only. This paper reviews four major methods of 3D tolerance analysis and compares them based on the literature published over the last three decades or so. The methods studied are Tolerance-Map (T-Map), matrix model, unified Jacobian–Torsor model and direct linearization method (DLM). Each of them has its advantages and disadvantages. The T-Map method can model all of tolerances and their interaction while the mathematic theory and operation may be challenging for users. The matrix model based on the homogeneous matrix which is classical and concise has been the foundation of some successful computer aided tolerancing software (CATs), but the solution of constraint relations composed of inequalities is complicated. The unified Jacobian–Torsor model combines the advantages of the torsor model which is suitable for tolerance representation and the Jacobian matrix which is suitable for tolerance propagation. It is computationally efficient, but the constraint relations between components of torsor need to be considered to improve its accuracy and validity. The DLM is based on the first order Taylor's series expansion of vector-loop-based assembly models which use vectors to represent either component dimensions or assembly dimensions. Geometric tolerances are operated as dimensional tolerances in DLM, which is not fully consistent with tolerancing standards. The results of four models with respect to an example are also listed to make a comparison. Finally, a perspective overview of the future research about 3D tolerance analysis is presented.

Contents

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1. Introduction

The objective of tolerance analysis is to check the feasibility and quality of assemblies or parts for a given GD&T scheme. The results of tolerance analysis include worst case variations and statistical distribution of functional requirement, acceptance rates, contributors and their percent contributions, and the sensitivity coefficients with respect to each contributor. Tolerance analysis is an essential part for mechanical design and manufacturing because it affects not only the performance of products but also the cost.

Tolerance analysis, including tolerance representation and tolerance propagation (tolerance transfer), can be classified into many categories based on the analysis objective and analysis approach, as shown in Fig. 1. According to dimensionality, there are one dimensional (1D), two dimensional (2D) and three dimensional (3D) tolerance analyses. Three approaches are applied for 1/2/3D tolerance analysis, i.e., worst case (deterministic case), statistical case and Monte Carlo simulation. Rigid and flexible tolerance analysis are two different models in the light of analysis objective. The former is surface-based and needs shape closure only, such as engines' tolerance analysis; the latter is point-based and needs shape and force closure simultaneously, such as auto-bodies' tolerance analysis where the finite element method (FEM) is used to take the deformation into consideration [1–4]. The division into part level and assembly level is another classification. The stack-up effect of assembly can be described by virtue of assembly function explicitly or implicitly, depending on the assembly method and sequence, as well as the property of components [5]. Tolerance analysis runs through the whole process of the product, including design, process planning, manufacturing, inspection, but the objective may be different in each phase. For example, the tolerance scheme, i.e., conventional (parametric) and geometric tolerance will be selected and specified, and then tolerance analysis for functional requirement will be carried out in design phase. Meanwhile, besides manual analysis, computer aided tolerancing software (CATs), such as VisVSA®, 3DCS® and CETOL® are applied to tolerance analysis successfully [6–9]. To be sure, the classification of tolerance analysis will be more and more complicated with the development of mechanical design and manufacturing.

Over the last thirty years, a large amount of fundamental research efforts has been given to explore the mathematical basis for tolerance analysis. For tolerance representation, the models or concepts include variational geometry [10–12], variational class [13,14], virtual boundary [15,16], feasibility space [17,18], vectorial approach [19], virtual joints [20], degree of freedom (DOF) [21–23], Tolerance-Map (T-Map) [24,25], topologically and technologically related surfaces (TTRS) [26], infinitesimal matrix [27], matrix [28–30], small displacement torsor (SDT) [31,32], and proportioned assembly clearance volume (PACV) [33,34]. Similarly, for tolerance propagation, the approaches or methods consist of the linearization method [35], system moments [36,37], quadrature [38–40], reliability index [41,42], the Taguchi method [43,44], Monte Carlo simulations [45,46], network of zones and datums [47], kinematic formulation [48], the direct linearization method (DLM) [49,50], Jacobian matrix [51,52], state space [53,54], and the variational method [55]. It is worth noting that the partition of two categories mentioned above is approximate and based mainly on their strong suits, because there is no boundary between the tolerance representation and propagation for these models, such as the TTRS [56].

As new generations of tolerancing standards, i.e., ASME Y14.5-2009 [57] and ISO 1101 [58] were released and popularized, geometric tolerances are generally accepted as industry practices. The traditional 1/2D tolerance analysis models are insufficient to meet the ever-tightening and increasingly complex requirements of tolerance analysis in various fields [59]. More specifically, variations of a feature caused by geometric tolerances are three dimensional, which cannot be considered by 1/2D methods. Researchers and engineers need a new method that can analyze how those geometric tolerances are represented and propagated in three dimensional space urgently. It is the 3D tolerance analysis method. Let us take a combustion engine as an example, as shown in Fig. 2. The translational and rotational variations of piston accumulated by geometric and dimensional tolerances of crank-link parts have a significant impact on the compression ratio. In addition, tolerances of parts affect not only the dimensional quality of assembly, but also other qualities such as frictional work [60,61] and sealing. Finding out the mapping relationship of tolerance between parts and functional requirements and performance indexes is important to engine design. 3D tolerance analysis methods will offer a significant clue for understanding the role of every tolerance of parts in the variation stream (gray boxes in Fig. 2).

The 3D tolerance analysis is an innovative method which represents and transfers tolerance in 3D space. Geometric tolerances and dimensional tolerances, as well as the interaction between them in the tolerance zone can be taken into consideration by 3D tolerance analysis methods. Moreover, abundant results, i.e., the translational and rotational variations of target feature are obtained in these methods. Many models have been developed for 3D tolerance representation and propagation since 1990s. Portman [27] introduces a spatial dimensional chain where the individual error is represented as an infinitesimal matrix to model the tolerance propagation. Fleming [47] illustrates the geometric relationships by a network of zones and datums connected by arcs to which constraints are assigned. The effects of these constraints are calculated through the network between nodes. Rivest et al. [48] propose a kinematic formulation which exploits the kinematic character of a toleranced feature relative to its datum. These three methods are preliminary explorations of 3D methods. Laperrière and Lafond [20,51] use virtual joints for tolerance representation and the Jacobian matrix for tolerance propagation. Davidson et al. [24] present a T-Map representing all possible variations of size, position, form, and orientation for a target feature. Desrochers and Rivière [29] represent the variations of a feature with a displacement matrix and transfer them with a homogeneous matrix. An SDT model introduced by Clément et al. [31] uses six small displacement vectors to represent the position and orientation of an ideal surface in relation to another ideal surface in a kinematic way. Desrochers et al. [62] put forward a unified Jacobian–Torsor model which combines the advantages of the torsor model and the Jacobian matrix. Chase et al. [50] introduce a DLM based on the first order Taylor's series expansion of vector-loop-based assembly models which use vectors to represent either component dimensions or assembly dimensions. Some models mentioned above have been applied extensively by virtue of CATs.