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A compact shape descriptor for triangular surface meshes

Zhanheng Gao [a,*], Zeyun Yu [b], Xiaoli Pang [c]

[a] College of Computer Science and Technology, Jilin University, China
[b] Department of Computer Science, University of Wisconsin at Milwaukee, USA
[c] The First Hospital of Jilin University, China

HIGHLIGHTS

- A compact Shape-DNA is presented to describe the shape of a triangular surface mesh.
- Compact Shape-DNA is composed of low frequencies of DFT of processed Shape-DNA.
- The method reduces up to 97% space and time consumptions compared to Shape-DNA.

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ABSTRACT

Three-dimensional shape-based descriptors have been widely used in object recognition and database retrieval. In the current work, we present a novel method called compact Shape-DNA (cShape-DNA) to describe the shape of a triangular surface mesh. While the original Shape-DNA technique provides an effective and isometric-invariant descriptor for surface shapes, the number of eigenvalues used is typically large. To further reduce the space and time consumptions, especially for large-scale database applications, it is of great interest to find a more compact way to describe an arbitrary surface shape. In the present approach, the standard Shape-DNA is first computed from the given mesh and then processed by surface area-based normalization and line subtraction. The proposed cShape-DNA descriptor is composed of some low frequencies of the discrete Fourier transform of the processed Shape-DNA. Several experiments are shown to illustrate the effectiveness and efficiency of the cShape-DNA method on 3D shape analysis, particularly on shape comparison and classification.

1. Introduction

With rapid generation and increasingly availability of digital models in recent years, surface shape analysis has become one of the most important tasks in computer graphics community [1]. Some popular applications are shape comparison, classification and retrieval. The problem of rigid shape comparison and retrieval has been well studied and a large number of methods and tools have been developed [2,3]. How to efficiently and accurately retrieve non-rigid (deformable) shapes from large databases, however, still remains a challenging problem, which inspires researchers to find good descriptors for non-rigid surface shapes. The existing methods on non-rigid shape descriptors can be roughly classified into two categories: global methods and local methods. Global methods use some global and isometric-invariant properties of shapes while local methods use local features of shapes as shape descriptors. We refer the readers to [4–7] for more details on these descriptors. The present paper is focused on the global methods and a new global and compact descriptor is proposed to efficiently describe shapes. Among the work on non-rigid shape description using global features, spectral-based methods have gained a lot of attention due to its representing simplicity and computational efficiency [8], and have been studied both theoretically [9] and computationally [10]. For a detailed survey of spectrum-based mesh processing and shape description, the readers are referred to [11].

Thanks to the property of isometric invariance, the Laplace–Beltrami (L–B) operator on a manifold has become one of the most popular operators for non-rigid shape analysis in such applications as matching [12], recognition [13–15], retrieving [16–18], segmentation [19] and registration [20]. In particular, the eigenvalues and eigenfunctions of the L–B operator play important roles in describing shapes for shape-based retrieving and mesh segmentation. Xu [21] proposed several schemes for discretizing the L–B

* Corresponding author. Tel.: +86 13504477391.
E-mail addresses: gzh@jlu.edu.cn (Z. Gao), yuz@uwm.edu (Z. Yu).

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operator on triangular meshes and established the convergence under various conditions. Brandman [22] approximated the eigenvalues of the L–B operator by solving an eigenvalue problem in a bounded domain, discretized into a Cartesian grid. Rong et al. [23] used the eigenvalues and eigenfunctions of the L–B operator for mesh deformation. Wu et al. [6] proposed a symmetric mean-value L–B operator and used it as a descriptor in 3D non-rigid shape comparison. Shi et al. [24] presented a surface reconstruction method based on the eigen-projection and boundary reformation of the L–B operator. Ruggeri et al. [3] described a method of matching 3D shapes based on the critical points of the eigenfunctions corresponding to some small eigenvalues of the L–B operator. As the eigenvalues are often computed on a mesh, a discrete approximation of the true underlying manifold, Dey et al. [25] studied the convergence and stability of eigenvalues to the true spectrum of the manifold. In addition to the traditional use for surface shapes, the L–B operator has been used for the recognition, retrieval and matching of images as well. Some early work dealing with those topics can be found in [26,27], in which the images are treated as Riemannian manifolds and the L–B or weighted L–B operators are applied to the manifold for characterizing the images.

From the perspective of signal processing, the eigendecomposition of the L–B operator can be thought of as an frequency analysis of the shape: the eigenvalues correspond to the frequency values and the eigenfunctions correspond to the signals of the associated frequencies. The Shape-DNA [28–31] consists of the N smallest eigenvalues of the L–B operator and is often used as a shape descriptor for measuring the similarity between different shapes by using the Euclidean (L2) distance between the Shape-DNA vectors. The property of isometric invariance derived from the L–B operator is one of the most important advantages of the Shape-DNA method, which makes it well suited for comparing non-rigid shapes. However, it is unclear as to what number of eigenvalues, i.e. N, should be used to form the Shape-DNA [32]. Reuter et al. used 20 eigenvalues for shape retrieval in [12] and 11 eigenvalues in [33]. In [34], the authors mentioned that 500 eigenvalues had to be computed for extracting important information from Dirichlet eigenvalues. However, in [35], the authors reported that 10–15 eigenvalues were enough for shape retrieving. In view of signal processing, more eigenvalues contains more information of detail and can describe the shape more accurately, but in the meantime, more time and space have to be used for computing, storing and comparing the Shape-DNAs. In this paper, we use at most 100 eigenvalues in the Shape-DNAs and our experiments show that the first 100 eigenvalues are typically enough for describing shapes in the database we used for testing.

Motivated by the Shape-DNA technique, we present a novel shape descriptor, called compact Shape-DNA (cShape-DNA), for analyzing the shape of a triangular surface mesh. The proposed method is a combination of the original Shape-DNA and discrete Fourier transformation (DFT), which encodes most of the shape information into only a small number of feature values and inherits all the advantages of the original Shape-DNA, including the isometric invariance. The time for computing the cShape-DNA is close to that of the original Shape-DNA, but the proposed shape descriptor requires smaller space for storing the cShape-DNA and less time for shape comparison, which makes the cShape-DNA a good candidate for fast shape retrieval especially in very large database applications.

The remainder of this paper is organized as follows. In Section 2, we introduce the cShape-DNA and the algorithmic detail. The comparison between the cShape-DNA and the original Shape-DNA for shape comparison and classification is made in Section 3. The impact of choosing different parameters and some other factors, such as noise and quality of the surface meshes, is also discussed in Section 3. The conclusion is given in Section 4.

2. Method

In this section, we first briefly review the original Shape-DNA and its computational procedure for a triangular surface mesh. We then elaborate on the detail of the proposed cShape-DNA.

2.1. The original Shape-DNA

Generally speaking, the Laplace–Beltrami (L–B) operator is the Laplace operator on a Riemannian manifold. It is defined as the divergence of the gradient of a function f which is defined on the manifold [36,37]:

$$\Delta f = \mathrm{div}(\mathrm{grad}(f)). \tag{1}$$

The eigenvalue problem of the L–B operator has the following form:

$$\Delta f = -\lambda f. \tag{2}$$

The solutions λ_i and f_i for $i = 0, 1, \ldots$ are called the eigenvalues and eigenfunctions of the L–B operator, respectively.

Let $\mathcal{M}$ be a triangular surface mesh in $\mathbb{R}^3$ with a set of vertices: $\mathcal{V} = \{\mathbf{v}_i\}_{i=1}^{N_\mathcal{V}}$. The eigenvalues of the L–B operator on $\mathcal{M}$ can be numerically computed by solving the following generalized eigenvalue problem:

$$A\mathbf{f} = -\lambda B\mathbf{f}, \tag{3}$$

where λ and $\mathbf{f}$ are considered unknown with $\mathbf{f} \triangleq \{f(\mathbf{v}_i)\}_{i=1}^{N_\mathcal{V}}$ being a vector of scalar function values $f(\mathbf{v})$ defined on the vertices of $\mathcal{M}$. The calculations of the $N_\mathcal{V} \times N_\mathcal{V}$ matrices, A and B, are detailed below. The obtained λ's and $\mathbf{f}$'s are the eigenvalues and the eigenfunctions of the L–B operator on $\mathcal{M}$ respectively, and the N smallest eigenvalues are known as the Shape-DNA of $\mathcal{M}$ [29,30].

The matrices A and B in Eq. (3) can be formulated when solving the partial differential equation in Eq. (2) with the finite element method (FEM), in which linear or higher order elements may be used. Although using quadratic or cubic elements typically yields better computational accuracy, the time cost for solving the corresponding FEM problem is much more expensive. After testing hundreds of mesh models taken from the McGill database [38], we choose to adopt the linear elements in our method because it yields almost identical Shape-DNAs to those obtained using quadratic or cubic elements but consumes much less time. With the linear finite element method, the matrices A and B take the following form when $\mathcal{M}$ is a closed mesh [33]:

$$a_{ij} = \begin{cases} \dfrac{\cot\alpha_{ij} + \cot\beta_{ij}}{2}, & \mathbf{v}_i\mathbf{v}_j \text{ is an edge in } \mathcal{M} \\ -\displaystyle\sum_{k\in N(i)} a_{ik}, & i = j \\ 0, & \text{other} \end{cases} \tag{4}$$

$$b_{ij} = \begin{cases} \dfrac{|t_1| + |t_2|}{12}, & \mathbf{v}_i\mathbf{v}_j \text{ is an edge in } \mathcal{M} \\ \displaystyle\sum_{k\in N(i)} \dfrac{|t_k|}{6}, & i = j \\ 0, & \text{other} \end{cases} \tag{5}$$

where t_1 and t_2 are the two triangles adjacent to edge $\mathbf{v}_i\mathbf{v}_j$, $|t_i|$ is the area of triangle t_i, α_{ij} and β_{ij} are the angles opposite to $\mathbf{v}_i\mathbf{v}_j$ in t_1 and t_2 respectively, and $N(i)$ is the index set of the vertices adjacent to $\mathbf{v}_i$.

The eigenvalues of the L–B operator is discrete and can be sorted in an increasing order: $\lambda_0 \le \lambda_1 \le \lambda_2 \le \cdots$. The first eigenvalue λ_0 is always 0 when $\mathcal{M}$ is closed.