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# Flat-end cutter orientation on a quadric in five-axis machining☆

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## HIGHLIGHTS

- The conditions to avoid local gouging and rear gouging are formulated.
- The machined strip width is evaluated analytically.
- Two orientation angles are fully exploited to maximise the width without gouging.
- The theory has been successfully applied in 5-axis sculptured surface machining.

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## ABSTRACT

The authors have recently developed methods for cutter orientation and tool path generation in 5-axis sculptured surface machining, where the design surface is approximated locally by a quadric. This paper presents, from a purely geometric perspective, the fundamental theory for optimising the cutter orientation on a quadric, which maximises the machined strip width whilst avoiding local and rear gouging. The analysis focuses on the flat-end cutter which is modelled by a circular cylinder but can be generalised for any fillet-end cutter using an appropriate offset of the design surface and the concept of geometric equivalency. The theory is illustrated by three examples.



## 1. Introduction

Five-axis machining is used widely in the aerospace, automobile and die/mould industries where machining efficiency and quality are crucial. In order to reduce cutting time and obtain a well-finished surface, an optimal cutter orientation is required. Much research effort has been concentrated on optimising the cutter orientation [1]. Recent developments of the quadric method (QM) [2] for orienting the cutter at one single point and the integrated method (IM) [3] for orienting the cutter along tool passes have significantly improved the machining efficiency and quality. In the two methods, there is a purely geometrical problem of how to orient a circular cylinder on a general quadric, which is addressed in this paper. Interestingly, the geometric interactions of cylinders and quadrics are encountered in many other applications such as engineering tribology [4], robotics [5] and rigid body simulations [6].

### 1.1. Basic concepts

To avoid any ambiguity, the basic concepts of cutter orientation, gouging, and machined strip width are introduced in this subsection.

*Cutter orientation* is used to specify how a flat-end cutter **C** is placed relative to a design surface **S** at their contact point, the *cutter contact (CC) point*. The orientation can be described in a *local machining coordinate system* (LMCS, $X_M$-$Y_M$-$Z_M$) as shown in Fig. 1. The origin is at the CC point. The $Z_M$-axis is along the surface

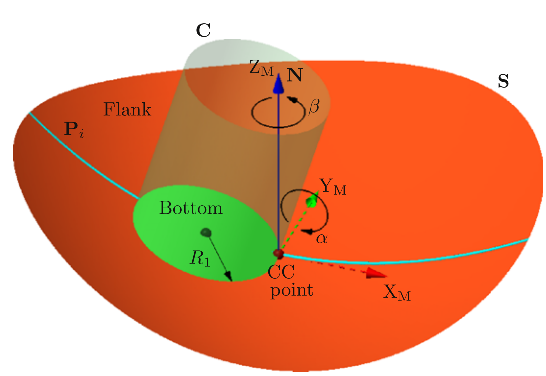


**Fig. 1.** Cutter orientation.

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normal **N** and the $X_M$-axis is in the *cutting direction*, which is tangent to a tool pass $\mathbf{P}_i$ at the CC point. The cutter is rotated first a *lead angle* $\alpha$ around the $Y_M$-axis, and then a *screw angle* $\beta$ around the $Z_M$-axis. In general, $0 \le \alpha \le \pi/2$, $-\pi/2 < \beta < \pi/2$, but the ranges may be further restricted by gouging avoidance between **C** and **S**.

Gouging is a critical problem in 5-axis machining. According to where gouging occurs on **C**, it is classified either as local gouging, rear gouging or global gouging (collision) [7]. In the paper, only local gouging and rear gouging are considered. *Local gouging* refers to the removal of excess material in the vicinity of the CC point, and is due to the mismatch in curvatures between **S** and the *cutter swept surface* $\mathbf{W}_i$, the envelope swept out by **C** along the tool pass $\mathbf{P}_i$. *Rear gouging* refers to the interference between **S** and a region on the bottom of **C** at some distance from the CC point.

Let an *error surface* **E** denote the offset of **S** with a distance equal to the machining tolerance $\varepsilon$ as shown in Fig. 2(a). Then, to ensure that *machining deviation*, the distance from the machined surface to **S**, is smaller than $\varepsilon$, the machined surface must lie between **E** and **S**. Define the *machined region* as an area on **E** bounded by intersection curves with **C**. Then on the tangent plane of **S** at the CC point as illustrated in Fig. 2 (b), the maximum span of the projected machined region along the $Y_M$-axis is the *machined strip width* $w$. Evidently, to improve machining efficiency at the CC point along $\mathbf{P}_i$, an optimal orientation is desired to maximise $w$ and give no gouging.

### 1.2. Previous work

There has been much research effort to optimise the cutter orientation. The published algorithms can be classified broadly into local and global methods. In local methods, only the normal curvatures of **C** and **S** at the CC point are considered to determine the orientation. **C** is inclined in the minimum principal direction of **S** [8,9] or in the cutting direction [10]. Then the lead angle $\alpha$ is determined to match the normal curvatures of **C** and **S** in the direction perpendicular to the cutting direction on the tangent plane. Rao and Sarma [11] and Yoon et al. [12] have improved the methods by comparing normal curvatures of $\mathbf{W}_i$ and **S** to avoid local gouging, but there might still be rear gouging.

To avoid the possibility of rear gouging, the global methods use an area of **S** beneath **C** to orient **C**. In the multi-point machining method [13,14], several contact points between **C** and **S** are found to determine the orientation. However, the method has to evaluate intersections between **C** and **S** iteratively, and there can be divergence in some cases [15]. In the arc-intersect method [16], $\alpha$ is set to the maximum rotation angle from sampled points on **S** to **C** when $\alpha = 0$. The weakness of the method is the low computational efficiency, since a bisection method is used at each sampled point for evaluating the rotation angle. Hosseinkhani et al. [17] developed the penetration–elimination method to avoid the angle evaluation, but a numerical root finding algorithm has to be applied at each point for gouging check.

To simplify the computation, another approach in global methods is to locally approximate **S** by a simple surface and then orient **C** on the simple surface. Based on the study of 5-axis spherical surface machining [18], Gray et al. [15] developed a rolling ball method which approximates **S** by part of a sphere and then places **C** on it. The method is easy to implement but, due to the limited shape defining capability of a sphere, the lead angle $\alpha$ is conservative in the sense that it could be reduced to generate a wider machined strip width [16]. Yoon [19] proposed a method to determine the orientation on an osculating paraboloid and then used the method for machining **S**, but there is a tacit assumption that **S** can be closely approximated by its second-order Taylor expansion at the CC point.

A more general approach involves the orientation of the flat-end cutter on an osculating quadric, which is the second-order algebraic surface form embracing all spheres, cylinders, cones, ellipsoids, paraboloids and hyperboloids [20]. It has been successfully applied in the quadric method (QM) [2] and integrated method (IM) [3], where practical products were machined and numerical simulations showed higher efficiency than previously published methods. To date, however, the geometric details for optimising the cutter orientation on a quadric have not been reported.

### 1.3. Outline of paper

This paper presents the fundamental theory for orienting a flat-end cutter on a quadric in 5-axis machining, to maximise the machined strip width and avoid local and rear gouging. The study is limited to orienting a cutter **C** on a design surface **S** at a single location under the following assumptions:

- **S** is a quadric.
- **C** is a flat-end cutter and its spindle speed is much higher than the feed rate, so it can be considered as a circular cylinder.
- The size of **C** has been selected appropriately [21].
- Potential collisions on the flank of **C** have been avoided by restricting the cutter orientation, but possible interference on the bottom of **C** needs to be checked.
- The allowable deviation of the machined surface from **S**, which is referred to as *machining tolerance* $\varepsilon$, is positive, and consistent with that in the finishing stage.

The conditions to avoid local gouging and rear gouging are formulated in Sections 2 and 3 respectively, followed by the evaluation of the machined strip width in Section 4. Then the two cutter orientation angles are optimised in Section 5 with three typical examples in Section 6 and an error analysis in Section 7. Finally, conclusions and future research directions are presented in Section 8.

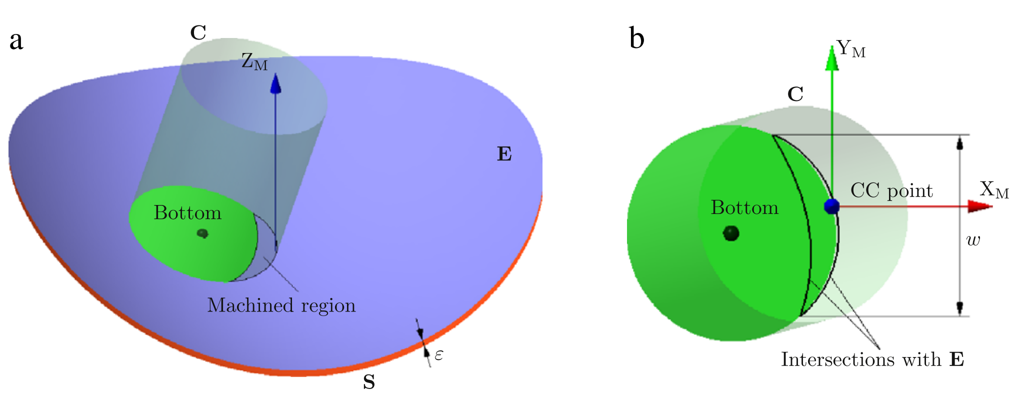


**Fig. 2.** Machined region (a) and machined strip width (b).