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# Robust reconstruction of 2D curves from scattered noisy point data[☆]

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## HIGHLIGHTS

- We propose a robust 2D reconstruction method from unorganized noisy point data.
- The outliers and noise of data can be effectively detected and smoothed.
- Sharp corners are preserved properly in the output curves with our method.

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## ABSTRACT

In this paper, a robust algorithm is proposed for reconstructing 2D curve from unorganized point data with a high level of noise and outliers. By constructing the quadtree of the input point data, we extract the "grid-like" boundaries of the quadtree, and smooth the boundaries using a modified Laplacian method. The skeleton of the smoothed boundaries is computed and thereby the initial curve is generated by circular neighboring projection. Subsequently, a normal-based processing method is applied to the initial curve to smooth jagged features at low curvatures areas, and recover sharp features at high curvature areas. As a result, the curve is reconstructed accurately with small details and sharp features well preserved. A variety of experimental results demonstrate the effectiveness and robustness of our method.


## 1. Introduction

Curve reconstruction from 2D point data is a fundamental problem in geometric modeling, reverse engineering, computational geometry, computer graphics and image processing, computer vision. For instance, in reverse engineering, one of the effective methods to model point data for fabrication using rapid prototyping techniques is to adaptively slice the data points, along a specific direction, into a series of layers and the points in each layer are treated as planar. By reconstructing the planar curve of each layer, the final model can be created using sweep or loft modeling operations [1,2].

Generally, curve reconstruction can be defined to compute curves to approximate the boundary point data as closely as possible. Over the past two decades, a number of curve reconstruction algorithms have been proposed [3–11]. In spite of considerable advances, there are still some problems with those methods, especially when the input point data contain a high level of noise and outliers. Furthermore, if sharp features (e.g. corners) exist within the curve, the requirement of being resilient to noise is particularly challenging since noise and sharp features are ambiguous, and most current techniques tend to blur out those sharp features or even amplify noisy samples.

In this paper, we present an effective curve reconstruction algorithm, where the input point data consist of a set of unorganized points around curve boundaries, ridden by a high level of outliers and noise. Specifically, by constructing the quadtree of the input point data, we extract the "grid-like" boundaries of the quadtree, followed by applying a modified Laplacian method to smooth the boundaries. The skeleton is computed and thereby the initial curves are constructed by circular neighboring projection. The projection method may produce some jagged edges. Therefore, we exploit a normal-based processing method on the initial curves to smooth out bumpy features at low curvatures areas, and to recover sharp features at high curvature areas. In contrast to previous

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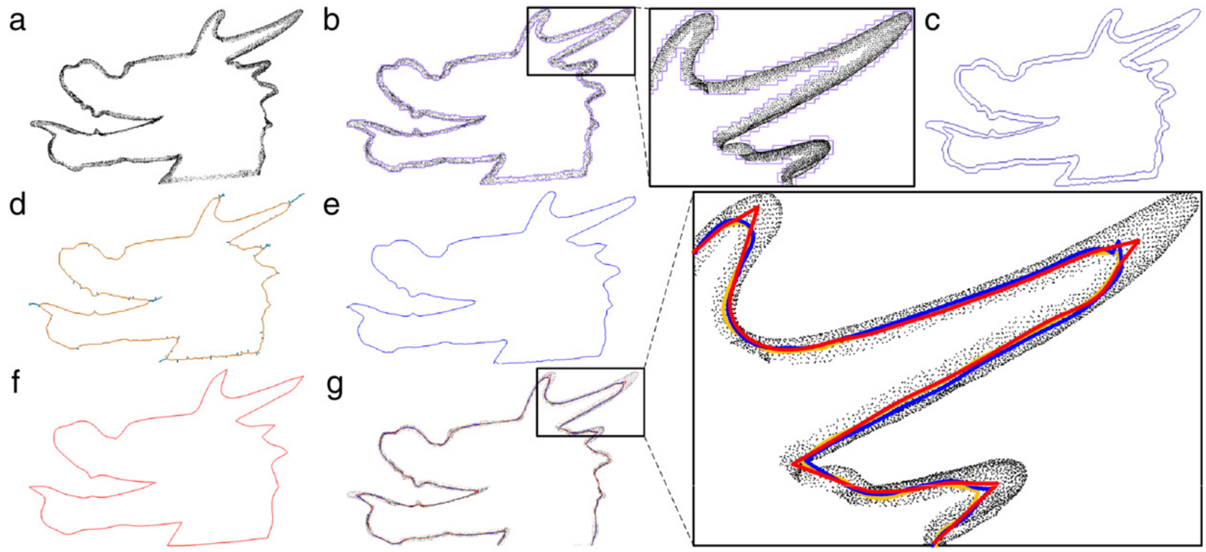


**Fig. 1.** Overview of our curve reconstruction pipeline. (a) The original point data; (b) The grid-like boundaries of the point data; (c) The smooth boundaries of the point data; (d) The skeleton of the smooth boundary, where the "*yellow*" curve is the main trunk, and the "*teal*" segments are short branches; (e) The initial curve reconstructed via circular neighboring projection; (f) The final reconstructed curve after normal-based smoothing; (g) The combination view of the original points, the main skeleton, the initial curve and the final curve. Note that sharp features are well recovered in the final curve. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

works, our method is capable of handling a high level of noise and outliers. Small details of the original curve can be recovered satisfactorily; meanwhile, sharp features (e.g. corners) are also nicely preserved. The pipeline is demonstrated in Fig. 1.

## 2. Related works

In this section, we briefly review the most related works on curve reconstruction from unorganized point data, and examine whether they have the ability to handle noise and outliers, and preserve sharp features.

Fang et al. [12] presented a method based on spring energy minimization to approximate unorganized point data with a curve. The nonlinear minimization problem of spring energy is solved by successive quadratic programming; however, this solution needs a good initial guess and priors of point data topology. Taubin et al. [13] designed a planar curve reconstruction method from unorganized point data using an implicit simplicial curve, defined by a planar triangular mesh and the values at the vertices of the mesh. These two methods are difficult to handle in cases with noise and outliers.

Pottmann et al. [14] used a pixel-based method to thin input point data to a curve, where the thinning technique is exploited to cope with noise. After defining an appropriate grid on the plane, pixels including one or more points are filled with black creating a binary image. Then the medial axis of the binary image is computed by using the image thinning algorithm. Finally, a smooth curve is achieved via curve approximation. Goshtasby [15] presented a method to compute a radial basis function surface on a point cloud, followed by discretizing the surface into an image. By tracing the spine of the image, the curve is achieved from the point cloud. In these methods, small branches of the median axis are ignored so that the small features of curves are filtered out and the reconstructed curve is consequently inaccurate.

The moving least squares (MLS) technique [16] is a powerful and robust point-set modeling approach. The basic idea is to search the neighbors of each point of the input data, and fit them by a curve with a weighted regression. The point is then replaced by the projection point on the curve. The procedure is repeated until the point data are thin enough to achieve the curve reconstruction. Note that the reconstruction result is dependent on the size of the selected neighbors. Lee [17] proposed a variant of the MSL method to reconstruct curves from unorganized point data, in which the size of neighbors is chosen based on the idea of principal component analysis. With this method, the noise is handled to some extent, but the sharp features are hard to be retained.

Poon [18] proposed an algorithm to reconstruct polygonal closed curves from noisy samples drawn from a set of smooth closed curves, which consists of three steps: point estimation, pruning and output. In the point estimation step, the noise is filtered out and new points are computed. A pruning step is taken to decimate the new points so that the interpoint distances in the pruned subset are large compared with their distances from the curve. Then, the NN-crust algorithm [4] is run in the output step to obtain the final curve. Lin et al. [19] reconstructed curves from non-uniformly sampling data based on an interval *B*-spline curve. The sequence joining method is exploited to cluster the point cloud into a rectangle sequence, and then two boundary point sequences are computed using the quasicentric point sequence. By fitting two boundary point sequences, an interval *B*-spline curve is obtained enveloping the strip-based point cloud. As a result, the noise of the points is filtered out. Wu et al. [20] designed an automatic reconstruction method of polygonal curves from unorganized dense planar points. The planar points are sorted, followed by decomposing the sorted points into different levels using *B*-spline wavelets. Then the polygonal curve is constructed hierarchically from coarser to finer level. From their experimental results, all those methods have good performance on *smooth* curve reconstruction from point data with a certain level of noise; however, they are incapable of preserving *sharp* features within curves, especially when the point data are highly noisy.

de Goes et al. [21] proposed a practical algorithm to address the problem of reconstruction and simplification of 2D curves from unorganized point sets based on an optimal transport technique. This method is able to robustly deal with feature preservation, like sharp intersections and corners. It also shows satisfactory robustness to a certain level of noise and outliers. However, it is hard to cope with heavy noise and outliers. In addition, small details from original curves generally are simplified by this method, and consequently the reconstruction results are relatively inaccurate. Our method takes into account all the input points within boundaries,