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Analysis-suitable volume parameterization of multi-block computational domain in isogeometric applications

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ABSTRACT

Parameterization of the computational domain is a key step in isogeometric analysis just as mesh generation is in finite element analysis. In this paper, we study the volume parameterization problem of the multi-block computational domain in an isogeometric version, i.e., how to generate analysis-suitable parameterization of the multi-block computational domain bounded by B-spline surfaces. Firstly, we show how to find good volume parameterization of the single-block computational domain by solving a constraint optimization problem, in which the constraint condition is the injectivity sufficient conditions of B-spline volume parameterization, and the optimization term is the minimization of quadratic energy functions related to the first and second derivatives of B-spline volume parameterization. By using this method, the resulting volume parameterization has no self-intersections, and the isoparametric structure has good uniformity and orthogonality. Then we extend this method to the multi-block case, in which the continuity condition between the neighbor B-spline volumes should be added to the constraint term. The effectiveness of the proposed method is illustrated by several examples based on the three-dimensional heat conduction problem.

1. Introduction

The isogeometric analysis (IGA for short) method proposed by Hughes et al. in [1] offers the possibility of bridging the gap between CAD and CAE. The approach uses the same spline representation both for the geometry and for the physical solutions, and thus avoids this costly back and forth transformations. This uniform framework provides more accurate and efficient ways to deal with complex shapes and to approximate the solutions of physical simulation problems. On the other hand, it also raises interesting geometric problems for analysis-suitable modeling tools [2–4].

It is well known that mesh generation, which generates a discrete mesh of a computational domain from a given CAD object, is a key and the most time-consuming step in finite element analysis (FEA for short). It consumes about 80% of the overall design and analysis process [5] in automotive, aerospace and ship industry. Parameterization of the computational domain in IGA, which corresponds to the mesh generation in FEA, also has some impact on the analysis result and efficiency. In particular, arbitrary refinements can be performed on the computational mesh in FEA, but in IGA if we compute with a tensor product B-splines, we can only perform refinement operations in each parametric direction by knot insertion or degree elevation. Hence, parameterization of the computational domain is also important for IGA. As it is pointed by Cottrell et al. [6], one of the most significant challenges towards isogeometric analysis is constructing trivariate spline volume parameterizations from a given CAD boundary representation.

From the viewpoint of graphics applications, volume parameterization of 3D models has been studied in [7–9]. As far as we know, there are only a few works on the parameterization of computational domains from the viewpoint of isogeometric applications. Martin et al. [10] proposed a method to fit a genus-0 triangular mesh by B-spline volume parameterization, based on discrete volumetric harmonic functions; this can be used to build computational domains for 3D IGA problems. Cohen et al. [11] proposed the concept of *analysis-aware modeling*, in which the parameters of CAD models should be selected to facilitate isogeometric analysis. They also demonstrated the influence of parameterization of computational domains by several examples. Escobar et al. proposed a method to construct a trivariate T-spline volume of complex genus-0 solids for isogeometric application [12]. However, the proposed method demands a surface triangulation as input data. A variational approach for constructing NURBS parameterization of swept volumes is proposed by Aigner

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et al. [13]. Given boundary CAD information, an approximate implicitization technique is used for parameterization of the 2D computational domain in [14]. In [15,16], the r-refinement method for generating optimal analysis-aware parameterization of the computational domain is proposed. However, it only works for specified analysis problems. A general construction method for analysis-suitable planar B-spline parameterization in the two-dimensional isogeometric problem is proposed in [17] based on harmonic mapping. In this paper, from the given boundary CAD information, the volume parameterization problem for the multi-block computational domain is studied based on the trivariate generalization of the method proposed in [16].

In IGA, the parameterization of a computational domain is determined by control points, knot vectors and the degrees of B-spline objects. For three-dimensional IGA problems, the knot vectors and the degree of the computational domain are determined by the given boundary surfaces. That is, given boundary surfaces, the quality of parameterization of the computational domain is determined by the positions of inner control points. Hence, finding a good placement of the inner control points inside the computational domain is a key issue. A basic requirement of the resulting volume parameterization for IGA is that it doesn't have self-intersections, so that it is an injective map from the parameterization domain to the computational domain. In order to get more accurate simulation results, the isoparametric structure in the computational domain should be as uniform as possible and have orthogonal isoparametric surfaces [18]. In this paper, we study the volume parameterization problem of the multi-block computational domain in the isogeometric version, i.e., how to generate analysis-suitable parameterization of the multi-block computational domain bounded by B-spline surfaces. Our main contributions are:

- A constraint optimization framework is proposed to generate a multi-block volume parameterization without self-intersections, and the resulting isoparametric structure has good uniformity and orthogonality;
- Some classical results in the field of differential geometry related to parametric surfaces are generalized to the case of trivariate parametric volumes, such as the orthogonal conditions of isoparametric structure and the C^1 conditions between B-spline volumes.
- We test the volume parameterization results on the heat conduction problem to show the effectiveness of the proposed method.

The remainder of the paper is organized as follows. Section 2 presents some preliminaries on B-spline volume parameterization. Section 3 describes the constraint optimization method for single-block volume parameterization of the 3D computational domain. Section 4 presents a volume parameterization framework for the multi-block computational domain based on the proposed methods in Section 3. Section 5 tests the volume parameterization results on the heat conduction problem to show the effectiveness of the proposed method. Finally, we conclude this paper in Section 6.

2. Preliminaries

For a parameterization σ from $\mathcal{P}\ :\ [a,b]\times[c,d]\times[e,f]$ to $\Omega\subset\mathbb{R}^3$, we define the boundary surfaces as the image of $\{a\}\times[c,d]\times[e,f]$, $\{b\}\times[c,d]\times[e,f]$, $\{c\}\times[a,b]\times[e,f]$, $\{d\}\times[a,b]\times[e,f]$, $\{e\}\times[a,b]\times[c,d]$, $\{f\}\times[a,b]\times[c,d]$ by σ. We say that σ defines a *regular boundary* if these surfaces do not intersect pairwise, except at their boundary curves and if they have no self-intersection points.

We consider the following trivariate B-spline parameterization

$$\sigma:(\xi,\eta,\zeta)\in\mathcal{P}:=[a,b]\times[c,d]\times[e,f]\mapsto\sigma(\xi,\eta,\zeta)$$
$$:=\sum_{\substack{0\le i\le l\\0\le j\le m\\0\le k\le n}}\mathbf{c}_{i,j,k}N_i^p(\xi)N_j^q(\eta)N_k^r(\zeta),$$

where $\mathbf{c}_{i,j,k}\in\mathbb{R}^3$ are the control points, and $N_i^p(\xi)$, $N_j^q(\eta)$ and $N_k^r(\zeta)$ are B-spline functions of degree p, q and r for a given knot vector on $[a,b]$, $[c,d]$ and $[e,f]$. Note that in this paper we use knot vectors with multiple end knots to force the volumes to interpolate the corner control points.

The derivative of $\sigma(\xi,\eta,\zeta)$ with respect to ξ can be expressed in terms of the differences $\Delta^1_{i,j,k}:=(\Delta^{1,x}_{i,j,k},\Delta^{1,y}_{i,j,k},\Delta^{1,z}_{i,j,k})=\mathbf{c}_{i+1,j,k}-\mathbf{c}_{i,j,k}$:

$$\partial_\xi\sigma=\sum_{\substack{0\le i\le l-1\\0\le j\le m\\0\le k\le n}}\omega^1_{i,j,k}\Delta^1_{i,j,k}N_i^{p-1}(\xi)N_j^q(\eta)N_k^r(\zeta),\tag{1}$$

where $N_i^{p-1}(\xi)$ is the B-spline function with one degree less in u, and $\omega^1_{i,j,k}$ is a positive factor.

Similarly, the derivative of $\sigma(\xi,\eta,\zeta)$ with respect to η and ζ can be expressed as follows

$$\partial_\eta\sigma=\sum_{\substack{0\le i\le l\\0\le j\le m-1\\0\le k\le n}}\omega^2_{i,j,k}\Delta^2_{i,j,k}N_i^{p}(\xi)N_j^{q-1}(\eta)N_k^r(\zeta),\tag{2}$$

$$\partial_\zeta\sigma=\sum_{\substack{0\le i\le l\\0\le j\le m\\0\le k\le n-1}}\omega^3_{i,j,k}\Delta^3_{i,j,k}N_i^{p}(\xi)N_j^{q}(\eta)N_k^{r-1}(\zeta),\tag{3}$$

where

$$\Delta^2_{i,j,k}=(\Delta^{2,x}_{i,j,k},\Delta^{2,y}_{i,j,k},\Delta^{2,z}_{i,j,k})=\mathbf{c}_{i,j+1,k}-\mathbf{c}_{i,j,k},$$
$$\Delta^3_{i,j,k}=(\Delta^{3,x}_{i,j,k},\Delta^{3,y}_{i,j,k},\Delta^{3,z}_{i,j,k})=\mathbf{c}_{i,j,k+1}-\mathbf{c}_{i,j,k},$$

and $\omega^2_{i,j,k}$, $\omega^3_{i,j,k}$ are positive factors.

2.1. Non-linear sufficient condition based on Jacobian computation

From [16], if the Jacobian determinant $\boldsymbol{J}(\sigma(\xi,\eta,\zeta))$ of trivariate B-spline parameterization satisfies $\boldsymbol{J}(\sigma(\xi,\eta,\zeta))>0$, then $\sigma(\xi,\eta,\zeta)$ has no self-intersections.

From (1)–(3) and the product properties of B-splines [19], the Jacobian determinant $\boldsymbol{J}(\sigma(\xi,\eta,\zeta))$ of the B-spline surface can be computed as follows:

$$\boldsymbol{J}(\sigma(\xi,\eta,\zeta))=\begin{vmatrix}\sigma^x_\xi&\sigma^x_\eta&\sigma^x_\zeta\\\sigma^y_\xi&\sigma^y_\eta&\sigma^y_\zeta\\\sigma^z_\xi&\sigma^z_\eta&\sigma^z_\zeta\end{vmatrix}$$
$$=\sum_{\substack{0\le i\le l-1\\0\le j\le m\\0\le k\le n}}\ \sum_{\substack{0\le i'\le n\\0\le j'\le m-1\\0\le k'\le n}}\ \sum_{\substack{0\le i''\le n\\0\le j''\le m\\0\le k''\le n-1}}N_i^{p-1}(\xi)N_j^q(\eta)N_k^r(\zeta)$$
$$\times N_{i'}^p(\xi)N_{j'}^{q-1}(\eta)N_{k'}^r(\zeta)\,N_{i''}^p(\xi)N_{j''}^q(\eta)N_{k''}^{r-1}(\zeta)$$
$$\times\,\omega^1_{i,j,k}\,\omega^2_{i',j',k'}\,\omega^3_{i'',j'',k''}\begin{vmatrix}\Delta^{1,x}_{i,j,k}&\Delta^{2,x}_{i,j,k}&\Delta^{3,x}_{i,j,k}\\\Delta^{1,y}_{i',j',k'}&\Delta^{2,y}_{i',j',k'}&\Delta^{3,y}_{i',j',k'}\\\Delta^{1,z}_{i'',j'',k''}&\Delta^{2,z}_{i'',j'',k''}&\Delta^{3,z}_{i'',j'',k''}\end{vmatrix}$$
$$=\sum_{i=0}^{3l-1}\sum_{j=0}^{3m-1}\sum_{k=0}^{3n-1}G_{ijk}N_i^{3p-1}(\xi)N_i^{3q-1}(\eta)N_k^{3r-1}(\zeta).\tag{4}$$

Hence, the Jacobian of trivariate B-spline parameterization can be represented in the form of trivariate B-spline volume with higher degrees. From the convex hull property of B-splines [20], we have the following theorem.