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Manufacture of a spur tooth gear in Ti–6Al–4V alloy by electrical discharge

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ABSTRACT

This paper proposes a method of manufacturing a spur tooth gear in Ti–6Al–4V alloy (grade 5) using a wire electrical discharge machine (Wire EDM). A geometrical model for the gear is drawn up and implemented using the program MATLAB.

The electro-erosion parameters tested for this alloy are applied to an ONA PRIMA S-250. The parameters used (power, pause, voltage, ...) are based on the ONA EDM charts. The Taguchi orthogonal array method was chosen to obtain the optimum values for cutting Titanium.

The work presented follows established lines for manufacturing mechanical parts using general purpose machines and tools. In this case, the WEDM process was used. The MATLAB program was employed to obtain the interpolation points. This program simplifies the task of solving the equations originated by the mathematical model which allows the wire path to be calculated.

The WEDM method used here is a commendable alternative for machining electrically conductible materials which are difficult to work with using conventional machine tools (milling, turning or boring). Furthermore, the WEDM process reduces or even eliminates the need for subsequent polishing processes due to the high-quality finish achieved.

1. Introduction

Wire electro-discharge machining (WEDM) is a manufacturing system for high precision cutting of complex shapes from hard metals or those which are difficult to cut using conventional machining methods (milling, turning, or drilling).

Two fundamental parameters govern the WEDM process: the power (electrical current) and the pause time. These parameters are usually tested by the machine manufacturer and are given in the instructions [1]. Materials exist, however, with which the user will have to experiment. Several authors have studied this subject, and their results show that the Taguchi orthogonal array method [2] is the most successful and it was chosen for this work in order to obtain optimum values for machining a titanium alloy by electro-discharge.

The purpose of this paper is to obtain the equation of the geometrical conversion of the involute curve for lineal segments (G1 numerical control order). This allows for the manufacture of gears using very hard materials which are difficult to work with on a gear generator machine. The example used here is the aeronautical titanium alloy Ti–6Al–4V.

The machines with numerical control that have only the basic functions of interpolation G1, G2 and G3, as in the case of the machines used here, are unable to carry out an involute profile. However, using the method described here it is possible to approach any curve (in this case, the involute) with sufficient precision [3]. The article indicated in Reference [4] was the inspiration for this work.

Instead of developing a specific procedure for the resolution of a mathematical model, it was decided to use a program for mathematical analysis such as MATLAB [5]. This program resolves the outlined equations, as well as generating the corresponding file, in which only G1 functions are used (linear interpolation), to transmit to the numerical control of the electro-discharge machine, which obtains the programmed profile of the gear with more than satisfactory precision.

2. Involute profile generation

An involute profile is a gear tooth profile, from the base circle to that of the addendum [6].

A parametric curve equation can be deduced from Fig. 1. See Reference [7].

$$\begin{aligned} x &= r_b \cdot \cos\alpha + \alpha \cdot r_b \cdot \sin\alpha \\ y &= r_b \cdot \sin\alpha - \alpha \cdot r_b \cdot \cos\alpha . \end{aligned} \quad (1)$$

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Nomenclature

List of symbols

α	Angle between the base circle radius and OX axis
$\alpha_{\max}$	Maximum value angle α
a, b, c	Normal straight line equation parameters
d	Chordal error
d_b	Base circle diameter
d_e	Addendum circle diameter
d_i	Dedendum circle diameter
d_p	Pitch diameter
ε	Angle between radii to end involute over addendum circle
ε_i	Initial value of angle ε with respect to the OX axis
ε_f	End value of angle ε with respect to the OX axis
e	Distance from a point to the symmetry axis
f	Wire feed
k	Correction factor
l	Wire travel length
m	Module
Q	Removed material flow
r	Generic radius (Addendum, Dedendum...)
r_a	Fillet radius
r_b	Base circle radius
r_e	Addendum circle radius
r_h	Wire radius
r_i	Dedendum circle radius
r_p	Pitch radius
V	Volume of eroded material
X	MATLAB vector that contains the coordinates $X(1) = x_1, X(2) = x_2, \ldots, X(i) = x_i, \ldots$
Y	MATLAB vector that contains the coordinates $Y(1) = y_1, Y(2) = y_2, \ldots, Y(i) = y_i, \ldots$
x, y	Involute coordinate. Also x, y are used in the line equation.
x_1, y_1	Known point involute A
x_2, y_2	Involute point to be found B
x_3, y_3	Involute point which must be d distance from segment AB.
Xc	MATLAB vector that contains the coordinates $Xc(1) = xc_1, Xc(2) = xc_2, \ldots, Xc(i) = xc_i, \ldots$
Yc	MATLAB vector that contains the coordinates $Yc(1) = yc_1, Yc(2) = yc_2, \ldots, Yc(i) = yc_i, \ldots$
x_c, y_c	Wire center coordinates
x_r, y_r	Rotate x, y coordinates
x_s, y_s	Symmetric x, y coordinates
x_i, y_i	Sequence of involute points
$y(x)$	Involute function
z	Number of teeth
$\Delta\phi$	Polygonal angle that interpolates the circle with a chordal error
β	Fillet arc angle end
ϕ	Angle variation interval for circular interpolation
γ	Polar radius angle of involute intersection point with pitch circle and OX axis
φ	Base radius angle of involute intersection point with polar radius of the same point
θ	Axis of symmetry angle of the tooth gap and OX axis
ξ	Addendum radius angle of involute intersection point with polar radius of the same point
ψ	Polar radius angle of involute intersection point with addendum circle and OX axis

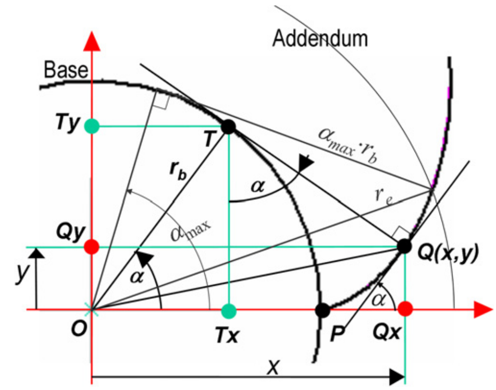

Fig. 1. Involute equation deduction and maximum α parameter.

The remaining involute curve points (x, y) can be obtained by assigning a range of values (in radians) to the α parameter [8].

As:

$$\frac{dy}{dx} = \tan\alpha \tag{2}$$

the angle between the straight tangent line to the involute and OX axis is then also α.

To obtain the interval of this parameter, the Pythagoras Theorem is applied to the triangle shown in Fig. 1, taking into account that the involute must reach the addendum circle and that the segment tangent to the base circle has the same length as the arc:

$$r_e^2 = r_b^2 + \alpha_{\max}^2 r_b^2 \Rightarrow \alpha_{\max} = \sqrt{\frac{r_e^2}{r_b^2} - 1}. \tag{3}$$

$$\text{Therefore } 0 \le \alpha \le \sqrt{\frac{r_e^2}{r_b^2} - 1}, \text{ expressed in radians.} \tag{4}$$

If $r_i > r_b$, the involute stops at the dedendum circle, and:

$$\sqrt{\frac{r_i^2}{r_b^2} - 1} \le \alpha \le \sqrt{\frac{r_e^2}{r_b^2} - 1}. \tag{5}$$

To design gear geometry, interference must be included [9] due to the difference between the theoretical diameter of the dedendum circle and the base circle, including a clearance.

$$\text{Interference: } d_i + \text{clearance} < d_b. \tag{6}$$

The critical tooth number can be determined from the fact that there is no involute at the part of the tooth profile below the base circle (see Fig. 2):

$$r_i + 0.25m < r_b \text{ or, } d_i + 0.5m < d_b \tag{7}$$

$$z \cdot m - 2.5 \cdot m + 0.5 \cdot m < z \cdot m \cdot \cos 20^\circ$$

$$z < \frac{2.5 - 0.5}{1 - \cos 20^\circ} \Rightarrow z < 33.1. \tag{8}$$

With less than 33 teeth, the diameter of the dedendum circumference is smaller than that of the base circle and requires a correction of the dentate, thus increasing the average diameter (which increases the distance between axes). However, for practical purposes a smaller number of teeth than the UNE norm [6], established at 30, is often considered. The correction factor by which it is necessary to multiply the module is given by the expression:

$$k = 0.03 \cdot (30 - z). \tag{9}$$