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Worst-case and statistical tolerance analysis based on quantified constraint satisfaction problems and Monte Carlo simulation

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ABSTRACT

This paper deals with the mathematical formulation of tolerance analysis. The mathematical formulation presented in this paper simulates the influences of geometrical deviations on the geometrical behavior of the mechanism, and integrates the quantifier notion (existential quantifier: "there exists"; universal quantifier: "for all"). It takes into account not only the influence of geometrical deviations but also the influence of the types of contacts on the geometrical behavior; these physical phenomena are modeled by convex hulls (compatibility hull, interface hull and functional hull) which are defined in parametric space. With this description by convex hulls, a mathematical expression of the admissible deviations of parts integrates the quantifier notion. This notion translates the concept that a functional requirement must be respected in at least one acceptable configuration of gaps (existential quantifier: "there exists"), or that a functional requirement must be respected in all acceptable configurations of gaps (universal quantifier: "for all"). To compute this mathematical formulation, two approaches based on Quantified Constraint Satisfaction Problem solvers and Monte Carlo simulation are proposed and tested.

1. Introduction

As technology improves and performance requirements continually tighten, the cost and the required precision of assemblies increase as well. There is a strong need for increased attention to tolerance design in order to enable high-precision assemblies to be manufactured at lower costs. Therefore, tolerance analysis is a key element in industry for improving product quality. To do so, a substantial amount of research has been devoted to the development of tolerance analysis. It can be either worst-case or statistical [1–4].

Worst-case analysis (also called deterministic or high–low tolerance analysis) involves establishing the dimensions and tolerances such that any possible combination produces a functional assembly, i.e. the probability of non-assembly is identically equal to zero. It considers the worst possible combinations of individual tolerances and examines the functional characteristic. Consequently, worst-case tolerancing can lead to excessively tight part tolerances and hence high production costs [2,4].

Statistical tolerancing is a more practical and economical way of looking at tolerances and works on setting the tolerances so as to ensure a desired yield. By permitting a small fraction of assemblies to not assemble or function as required, an increase in tolerances for individual dimensions may be obtained, and in turn, manufacturing costs may be reduced significantly [3]. Statistical tolerance analysis computes the probability that the product can be assembled and will function under a given individual tolerance.

This state of the art based on academic papers and also on four commercial systems (CATIA 3D FTA from Dassault Systèmes, CE/TOL 6 Sigma from Sigmetrix, e-TolMate from Tecnomatix and CAT_3DCS from DCS) points out a main difference between these commercial systems, the first two analyze one "sample" of an assembly and are based on a linear algebraic problem, whereas the later ones require a large number of "samples" to achieve reasonable accuracy and are based on statistics [2]. The models used within the systems are not clearly presented because it is very difficult to obtain information from CAT system vendors.

The analysis methods are divided into two distinct categories based on the type of accumulation input: displacement accumulation and tolerance accumulation.

- The aim of displacement accumulation is to simulate the influences of deviations on the geometrical behavior of the mechanism. Usually, tolerance analysis uses a relationship of the form [3]:

$$Y = f(X_1, X_2, \ldots, X_n) \quad (1)$$

where Y is the response (characteristics such as gap or functional characteristics) of the assembly and $X = \{X_1, X_2, \ldots, X_n\}$ are the values of some characteristics (such as situation deviations or/and intrinsic deviations) of the individual parts or subassemblies making up the assembly. The part deviations could be represented by kinematic formulation [5], small displacement torsor (SDT) [6], matrix representation [7], vectorial tolerancing [8] etc.

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The function f is the assembly response function which represents the deviation accumulation. The relationship can exist in any form for which it is possible to compute a value for Y given values of $X = \{X_1, X_2, \ldots, X_n\}$. It could be an explicit analytic expression or an implicit analytic expression. In a particular relative configuration of parts of an assembly consisting of gaps without interference between parts, the composition relations of displacements in some topological loops of the assembly permit determining the function f. For hyperstatic assembly, determination of function f is very complex, whereas this determination is easy for an open kinematic chain.

For statistical tolerance analysis, the input variables $X = \{X_1, X_2, \ldots, X_n\}$ are continuous random variables which enable representing tolerances. In general, they could be mutually dependent. A variety of methods and techniques (Linear Propagation (Root Sum of Squares), Nonlinear propagation (Extended Taylor series), Numerical integration (Quadrature technique), Monte Carlo Simulation etc.) are available for estimation of the probability distribution of Y and the probability $P(T)$ with respect to the geometrical requirement [3].

- The aim of tolerance accumulation is to simulate the composition of tolerances i.e. linear tolerance accumulation and 3D tolerance accumulation. Based on the displacement models, several vector space models map all possible manufacturing variations (geometrical displacements between manufacturing surfaces or between manufacturing surface and nominal surface) into a region of hypothetical parametric space. The geometrical tolerances or the dimensioning tolerances are represented by deviation domain [9–11], T-Map® [12,13] or specification hull [14,15]. These three concepts are a hypothetical Euclidean volume which represents all possible deviations in size, orientation and position of features.

  For tolerance analysis, this mathematical representation of tolerances allows calculation of accumulation of the tolerances by Minkowsky sum of deviation and clearance domains [10,11]: to calculate the intersection of domains for parallel kinematic chain and to verify the inclusion of a domain inside other one. The methods based on this mathematical representation of tolerances are very efficient for the tolerance analysis.

However, these two approaches do not take into account the quantifier notion. This notion translates the concept that a functional requirement must be respected in at least one acceptable configuration of gaps (existential quantifier: "there exists"), or that a functional requirement must be respected in all acceptable configurations of gaps (universal quantifier: "for all") [16,15]. A configuration is a particular relative position of parts of an assembly consisting of gaps without interference between parts.

The quantifier notion impacts the result of the tolerance analysis [16,15]. Therefore, we propose a mathematical formulation of tolerance analysis which simulates the influences of geometrical deviations on the geometrical behavior of the mechanism, and integrates the quantifier notion. To compute this mathematical formulation, two approaches based on Quantified Constraint Satisfaction Problem solvers and Monte Carlo simulation are proposed and tested.

2. Quantifier notion and mathematical formulation of tolerance synthesis

In this section, the quantifier notion is illustrated with a geometrical requirement and with an assembly requirement.

2.1. Quantifier notion for geometrical product requirement

A mechanism is a set of parts with joints. Most of the joints have functional gap. These gaps induce displacements between parts. Each relative position defines a configuration of the joint. A configuration is a particular relative position of parts of an assembly consisting of gaps without interference between parts. The product geometrical requirement limits the variation between two surfaces of the mechanism, which are in functional relation. This requirement is a condition on the functional characteristic between these surfaces. For any given mechanism with gap [14,15], the relative orientation or position of these surfaces depends on the configuration, which is not single. Therefore, the value of the functional characteristic depends on the configuration of the mechanism. There is an ambiguity in the expression of the requirement because the considered configuration is not described. In order to address this problem, it is necessary to specify: in which configuration, the condition of the geometrical requirement must be checked. The expression of the geometrical product requirement is not univocal [16].

So, to define a univocal expression of the condition corresponding to a geometrical product requirement, this expression is completed by a quantifier ($\exists$ or $\forall$). The quantifier translates the concept that the condition must be respected in at least one configuration of the mechanism ($\exists$), or that the condition must be respected in all configurations of the mechanism ($\forall$).

- In the case of the quantifier $\exists$, **if there exists** one configuration of the mechanism **such that** the value of the functional characteristic is less than or equal to the tolerance, **then** the geometrical product requirement is respected.
- In the case of the quantifier $\forall$, **if for all** configurations of the mechanism, the value of the functional characteristic is less than or equal to the tolerance, **then** the geometrical product requirement is respected.

2.2. Mathematical formulation of tolerance analysis for geometrical product requirement

In the CAD environment, a model is represented by ideal dimensions known as nominal dimensions. The nominal dimension is the representation of the ideal representation of the part model geometry. Due to the variations associated with manufacturing process, it is not possible to attain this nominal dimension in a repetitive manner. In reality, any specific dimension might vary within a defined range due to reasons such as setup errors, tool wear and many other factors. In order to account for these factors and to ensure the desired behavior of the assembly in spite of variations, the component features are assigned a parametric zone within which the value of the feature i.e. situation and intrinsic lie.

The approach used in this paper is a parameterization of deviations from theoretic geometry, the real geometry of parts is apprehended by a variation of the nominal geometry. The substitute surfaces model these real surfaces. This parameterization of variations is detailed in Section 2.2.1, and it enables us to define a variations parametric space, in which each coordinate system axis represents a parametric variable.

The mathematical formulation of tolerance synthesis takes into account not only the influence of geometrical deviations on the geometrical behavior of the mechanism and on the geometrical product requirements, but also the influence of the types of contacts on the geometrical behavior; all these physical phenomena are modeled by convex hulls (compatibility hull, interface hull and functional hull; these convex hulls are detailed in Section 2.2.2) which are defined in the variations parametric space. A *convex hull* or a *convex polytope* [17,18] may be defined as a finite