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A physically based method for triangulated surface flattening

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Abstract

This paper introduces a new method of generating 2D flat patterns from a 3D triangulated surface by opening the bending configuration of each winged triangle pair. The flattening can be divided into four steps. First, a 3D triangulated surface is modeled with a mass–spring system that simulates the surface deformation during the flattening. Second, an unwrapping force field is built to drive the mass–spring system to a developable configuration through the numerical integration. Third, a velocity redistribution procedure is initiated to average velocity variances among the particles. Finally, the mass–spring system is forced to collide with a plane, and the final 2D pattern is generated after all the winged triangle pairs are spread onto the colliding plane. To retain the size and area of the original 3D surface, a strain control mechanism is introduced to keep the springs from over-elongation or over-shrinkage at each time step.

Keywords: Unwrapping force; Velocity redistribution; Collision; Strain control; Flattening

1. Introduction

3D surface flattening is a subject widely studied in the field of CAD and computational geometry. Flat patterns generated from original 3D configurations can be used for reverse engineering to approximate the 3D objects, and are also critical for creating a texture atlas. In fashion design and apparel manufacture, a garment is constructed through seaming 2D flat patterns. When the garment is dressed onto a human body, each piece of the pattern demonstrates a 3D shape that is topologically equivalent to a disk. Surface flattening deals with a reverse issue, i.e., to generate 2D patterns from the 3D garment and to ensure that each pattern has an accurate boundary that promises the expected 3D shape once it is sewn with another piece. The methods presented in this paper are intended for this matter.

If the original 3D object is represented by a triangulated surface, the boundary is a closed contour and consists of directional triangle edges. During the flattening, the distortion of edges contributes to the shaping of the boundary. Thus, a successful flattening method should be able to handle the distortion either individually or collectively, and usually the insertion of a seam/dart is inevitable.

The method presented in this paper is aimed at unfolding winged triangles that share the same edge by an unwrapping force field. Once most winged triangle pairs have been unfolded, the surface will be forced to collide with a given plane to generate the final flattened configuration. To retain the boundary fidelity and to minimize the distortion during unfolding, a strain control mechanism has been emphasized on each triangle edge. The whole procedure is a purely physical approach compared to the current published methods. Different from other physically based methods for surface flattening, our method does not need a one-to-one mapping procedure to obtain the initial flat pattern. The surface patch is considered as a shell material that is flattened by the internal and external forces.

2. Previous work

The methods based on surface parameterization for flattening a 3D surface involve decomposing the surface to discrete patches, building the correspondence between 3D meshes and their isomorphic counterparts in a 2D plane through piecewise mapping, and minimizing the introduced distortions via a linear and non-linear solver. An excellent overview of the recent advances in parameterization was given in [1]. The major

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concerns and differences of these methods concentrate on how to measure and minimize the distortion in parameterization. In the early paper of Eck et al. [2], the parameterization was performed according to the harmonic maps which require a definition of the mesh boundary for the best conformality. In the work of Levy et al. [3], the parameterization was achieved by satisfying the Cauchy–Riemann equation in each triangle, and no boundary definition was needed. Haker et al. [4] employed the rules of differential geometry for conformal mappings, and the boundary condition was also needed in their algorithm. Desbrun et al. [5] proposed a method to maintain both the conformality and authalicity during parameterization to minimize the distortion of different intrinsic measures of the original mesh. Floater [6] presented a method for making shape-preserving parameterizations of surface triangulations based on graph theory. Recently, Yoshizawa et al. [7] improved this method by optimizing the parameterization generated at the previous step to reach a low-stretch result. This is actually a redistribution of local stretches.

For the parameterization of a garment surface, the profile of the flattened pattern and the minimization of strain energy are of most concern. McCartney et al. [8] presented a method for flattening the candidate triangle sets iteratively while evaluating the local distortion energy of particle movements. Based on the same methodology, Wang et al. [9] refined this method by transferring the whole surface into a mass–spring system and applying a penalty function to recover the overlapped area. The deformation produced in the flattening procedure was dissipated through energy release and surface cutting. Recently, McCartney et al. [10] proposed an orthotropic strain model to mimic the flattening problem of orthotropic materials. The strain energy is minimized through a numerical gradient optimization technique based on the Broyden–Fletcher–Goldfarb–Shanno method. Wang et al. [11] proposed another method involving inserting the seeds in the discrete geodesic curve generation algorithm for 3D surface fitting, and then establishing the planar coordinate mapping between the 3D surface and its counterpart in the plane by geodesic interpolation of the mappings. Strain energy minimization was emphasized in both seed insertion and the mapping procedure.

A common feature of these methods is that they need to establish the 2D isomorphic planar configuration through a one-to-one geometrical mapping and then minimize the introduced distortion energy through various approaches. The mapping usually requires a specially designed data structure and rules to determine the best 2D coordinates. Different from this manner of flattening, our method is to transfer the 3D surface into an elastic shell which is actually represented by a mass–spring system, and to release the embedded bending via an unwrapping force field.

Mass–spring systems have been widely used in cloth simulation. In such a system, a sheet of fabric is discretized into mass particles networked by springs, and the deformation of the fabric is visualized through the movement of each particle governed by spring forces. If the mass of the particles and the stiffness and viscosity of the springs are selected properly, this model may realistically demonstrate cloth-like deformations for different types of fabrics with specific fiber contents and structures. Cloth simulation using mass–spring systems has been investigated for over two decades. More detailed reviews on this topic can be found in [16] and [17]. Other successful fabric modeling methods can be found in [12,15,18–24].

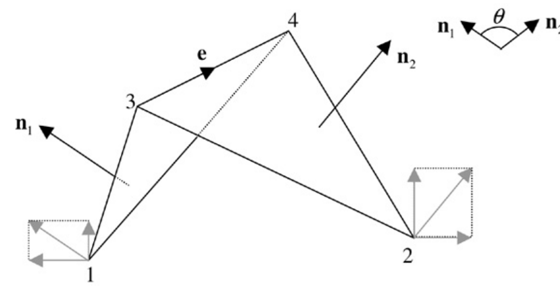

Fig. 1. An unfolding element with dihedral angle $\pi - \theta$.

An unconstrained mass–spring system is likely to produce “super-elastic” effects. Since most of the applications do not allow excessive distortions, the strain (elongation/compression rate) of the meshes should not exceed a given tolerance. Though some researchers thought that any edge of a triangular mesh should not change by more than 10% in a single time step [25], we set this tolerance to be 0.5%. The unwanted strain occurring in a spring can be offset by adjusting either the position [12] or the velocity [15] of the two end points of the spring. Both methods can effectively restrain the strain within the limit. A major difference is that the position adjustment has the possibility of introducing extra intersections between cloth meshes and the subject surface, or self-intersections among cloth meshes.

3. Flattening

3.1. Unwrapping (stage I)

The first step of our method is to open every two winged triangles in the original surface by applying an unwrapping force. It is defined as a combination of unfolding and spreading, which is illuminated in the work of Baraff and Witkin [22] and the work of Bridson et al. [14] in calculating the bending force.

As shown in Fig. 1, an unfolding element is a winged pair of triangles that consist of four particles. We use $\mathbf{x}$, $\mathbf{v}$ and $\mathbf{F}$ to denote the positions, velocities and unfolding forces of the particles. $\mathbf{n}_1$ and $\mathbf{n}_2$ are the normal of two winged triangle pairs, and $\mathbf{e}$ is the normalized vector from particle 3 to particle 4. If the original dihedral angle of the winged triangle pairs is $\pi - \theta$, the resulting dihedral angle is set as π to reflect the flattened position.

During the unwrapping movement, the velocities $\mathbf{v}_i$ $(i = 1, 2, 3, 4)$ and the unfolding forces $\mathbf{F}_i$ $(i = 1, 2, 3, 4)$ of the particles exist in a 12-dimensional linear space [14]. There are 12 distinct “modes” of motion for an unfolding element. The first 11 modes, which include three rigid (instantaneous) body translations, three rigid body rotations, two in-plane motions of vertex 1, two in-plane motions of vertex 2, and one in-line stretching of edge 3–4, do not affect the dihedral