Computer Aided Geometric Design 39 (2015) 50–66

Contents lists available at ScienceDirect

Computer Aided Geometric Design

www.elsevier.com/locate/cagd

# A 3D shape descriptor based on spectral analysis of medial axis ☆

Shuiqing He [a,*], Yi-King Choi [a], Yanwen Guo [b], Xiaohu Guo [c], Wenping Wang [a]

[a] *Department of Computer Science, The University of Hong Kong, Hong Kong*
[b] *National Key Lab for Novel Software Technology, Nanjing University, Nanjing, China*
[c] *Department of Computer Science, The University of Texas at Dallas, Richardson, TX, USA*

ARTICLE INFO

*Article history:*
Received 7 July 2015
Accepted 30 August 2015
Available online 9 October 2015

*Keywords:*
Spectral analysis
Laplace–Beltrami operator
Medial axis
Shape descriptor
Shape matching
Shape retrieval

ABSTRACT

The medial axis of a 3D shape is widely known for its ability as a compact and complete shape representation. However, there is still lack of a generative description defined over the medial axis directly which limits its actual application to 3D shape analysis such as shape matching and retrieval. In this paper, we propose a new spectral shape descriptor that directly applies spectral analysis to the medial axis of a 3D shape, which we call the *medial axis spectrum* for a 3D shape. We develop a newly defined Minkowski–Euclidean ratio inspired by the Minkowski inner product to characterize the geometry of the medial axis of a 3D mesh. We then generalize the Laplace–Beltrami operator to the medial axis, and take the solution to a Laplacian eigenvalue problem defined on it as the medial axis spectrum. The medial axis spectrum is invariant under rigid transformation and isometry of the medial axis, and is robust to shape boundary noise as shown by our experiments. The medial axis spectrum is finally used for 3D shape retrieval, and its superiority over previous work is shown by extensive comparisons.



## 1. Introduction

The medial axis of a shape comprises the set of interior points, each of which has two or more closest points on the shape boundary (Siddiqi and Pizer, 2008). Each point on the medial axis can also be associated with the local thickness, symmetry information and part-structure of the object, an advantage not possessed by boundary shape representation. This makes the medial axis based shape representation compact and complete in the sense that the original shape can be reconstructed from its medial axis. For this reason, the medial axis shape representation is widely used for shape editing and processing.

However, there is still lack of a generative description defined over the medial axis directly. This limits the actual application of the medial axis to 3D shape analysis. As a matter of fact, in shape indexing, matching and retrieval, the medial axis has not been fully exploited directly but is only used for derivation of a high-level graph abstraction to which graph matching or graph spectral techniques are applied (Shokoufandeh et al., 2005; Siddiqi et al., 1999, 2008).

☆ This paper has been recommended for acceptance by Rida Farouki.

* Corresponding author. Tel.: +852 95388869.
*E-mail addresses:* qingshui5889@gmail.com (S. He), ykchoi@cs.hku.hk (Y.-K. Choi), ywguo.nju@gmail.com (Y. Guo), xiaohuguo@gmail.com (X. Guo), wenping@cs.hku.hk (W. Wang).

http://dx.doi.org/10.1016/j.cagd.2015.08.004
0167-8396/

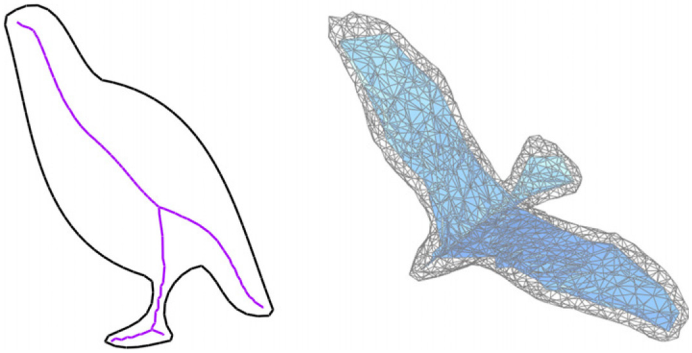

**Fig. 1.** Medial axes of a 2D bird (left) and a 3D eagle (right).

In this paper, we propose a new medial axis based spectral shape descriptor which we call the medial axis spectrum for a 3D shape. The eigenfunctions of the Laplace–Beltrami operator described by Levy (2006) have the capacity of understanding the global surface property of a shape. This can be used for obtaining an isometry-invariant descriptor known as the Shape-DNA (Reuter et al., 2006), based on the boundary shape representation. We show that the spectral analysis approach can be directly applied to the medial axes of 3D shapes. This yields a generative medial axis shape descriptor which could be used for shape analysis tasks including shape matching and retrieval.

Applying spectral analysis to the medial axis of a 2D shape has been explored by the previous work (He et al., 2014), which makes use of the derivative of medial axis radius function that describes the local geometry of the medial axis. It is, however, not intuitive to generalize it to a 3D shape. Fig. 1 shows a comparison between the medial axes of a 2D bird and a 3D eagle. The medial axis of a 2D shape is a 1D curve even though medial curves may connect at branch points. In contrast, the medial axis of a 3D shape is a surface in 3D. Although the dimension is just one degree higher, parametrization of the medial surface is not readily available. This makes it challenging to find a derivative similar metric on the radius function of a 3D shape for describing the local geometry of the medial surface, which needs to be fed to our derived Laplacian eigenfunctions. To this end, we develop a newly defined Minkowski–Euclidean ratio inspired by the Minkowski inner product to characterize the geometry of the medial axis surface of a 3D mesh. We then generalize the Laplace–Beltrami operator to the medial axis, and take the solution to a Laplacian eigenvalue problem defined on the surface as the medial axis spectrum.

The medial axis spectrum we propose has several favorable properties. On the one hand, it is invariant to rigid transformation of the input shape as well as isometry of the medial axis. On the other hand, the medial spectrum is robust to shape boundary noise and remains stable upon small perturbation of the medial axis. The medial axis itself is known for its instability. That is, small variations of object boundary may lead to a big fluctuation to the medial axis, a phenomenon that hinders severely the effective utilization of medial axes. We measure quantitatively the bound of the medial axis spectrum for the perturbed medial axis. Our experiments also show that the spectrum is resistant to the perturbation.

We will first review the related work in Section 2. We then present the formulation of the medial axis spectrum in Section 3 and the computation details with robustness analysis in Section 4. Experimental results and an application of the medial axis spectrum to shape retrieval will be given in Sections 5 and 6. We finally conclude the paper in Section 7.

## 2. Related work

### 2.1. Spectral shape analysis

Spectral shape analysis aims to capture the intrinsic properties of a shape by studying the spectrum of the Laplace–Beltrami operator of a shape. Invariant descriptors are thus derived. The pioneer work by Reuter and his colleagues (Reuter et al., 2006) proposed to use a subset of eigenvalues of the Laplace–Beltrami operator to serve as a shape descriptor, namely the Shape-DNA. The sequence of the first $k$ eigenvalues is also called the *k-spectrum*. The spectrum is invariant to translation of the shape and is isometric invariant. Additionally the spectrum can be normalized (Reuter et al., 2006) so that scaling factors for the geometric shape can be obtained easily (Reuter et al., 2005). It has been known that the $k$-spectrum is effective in distinguishing different shapes in practice, though in theory there exist different shapes (known as the isospectral shapes) yielding the same spectrum (Reuter et al., 2006). We follow the same spirit here: while the Shape-DNA method considers the Laplace–Beltrami operator on a shape boundary, we extend the operator to the medial axis of a 3D shape.

Rustamov (2007) presented the global point signature (GPS) embedding by combining the Laplace–Beltrami eigenvalues and eigenfunctions. This signature is robust to local topology changes. Both Shape-DNA and GPS are global shape descriptors. Sun et al. (2009) introduced the heat kernel signature, a multi-scale local shape descriptor which is insensitive to topological noise at small scales and can be used for partial shape matching. Due to the Laplace–Beltrami formulation, these shape descriptors are invariant under isometric deformation and therefore allow shape bending without distorting the local metric on a shape, which is commonly observed in natural articulated objects.