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ORIGINAL ARTICLE

Research of population with impulsive perturbations based on dynamics of a neutral delay equation and ecological quality system

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Abstract This paper studies the global behaviors of a nonlinear autonomous neutral delay differential population model with impulsive perturbation. This model may be suitable for describing the dynamics of population with long larval and short adult phases. It is shown that the system may have global stability of the extinction and positive equilibria, or grow without being bounded under some conditions.

1. Introduction

Many evolutionary processes in nature are characterized by the fact that their states experience abrupt changes at certain moments, which can be described by impulsive systems. Moreover, the impulsive systems have much richer dynamics than the corresponding non-impulsive systems. This is the reason that this paper studies the neutral delay equation for an insect population with impulsive perturbations.

In Stephen and Kuang (2009), we get

$$u'_m(t) = u_0(\tau - t)e^{-\mu t} - d(u_m(t))t \leqslant \tau \tag{1.1}$$

$$\begin{aligned} u'_m(t) = \big(b_2 u'_m(t-\tau) + b_2 d(u_m(t-\tau)) + b_0 u_i(\tau - t) \\ + b_1 u_m(\tau - t)\big)e^{-\mu t} - d(u_m(t))t \geqslant \tau \end{aligned} \tag{1.2}$$

Just like Stephen and Kuang (2009), let t and a denote time and age and let $u(t,a)$ be the density of individuals of age a at time t. It will be assumed that individuals take time τ to mature, so that the total numbers of mature and immature numbers u_m and u_i are given respectively by

$$u_m(t) = \int_\tau^\infty u(t,a)da, \quad u_i(t) = \int_0^\infty u(t,a)da$$

and with the initial condition $u_0 = u(0,a) \geqslant 0, \quad a \geqslant 0$ $u(t,0) = \int_0^\infty b(a)u(t,a)da$

Following Bocharov and Hadeler (2000), the birth rate function,

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$b(a) = b_0 + (b_1 - b_0)H(a-\tau) + b_2\delta(a-\tau)$

where $H(a)$ is Heaviside function and $\delta(a)$ the Dirac delta function. This choice for $b(a)$ implies that individuals age less than τ produce b_0 eggs per unit time, those of age greater than τ produce b_1 eggs per unit time, and additionally each individual lays b_2 eggs on reaching maturation age τ (the b_2 eggs all being laid at exactly that instant). In fact, we shall take $b_0 = 0$ for most of this paper, because most individuals do not lay eggs until they reach maturation age τ in nature.

In this paper, we will study the following system with impulsive perturbations, we will drop the subscript m for convenience,

$$\begin{cases} u'(t) = u_0(\tau - t)e^{-ut} - d(u(t)) & t \leqslant \tau \\ u'(t) = (b_2u'(\tau - t) + b_2d(u(t-\tau)) + b_1u(\tau - t))e^{-ut} - d(u(t)) & t \geqslant \tau \\ u(\tau_k^+) = (1+c_k)u(\tau_k) & k = 1,2,3,\ldots \end{cases}$$

We will use the following hypotheses:

(H1) $0 = \tau_0 < \tau_1 < \tau_2 < \ldots$ are fixed impulsive points with $\lim_{k\to\infty} = \infty$, $\tau_k = k\tau$.
(H2) (c_k) is a real sequence and $c_k > -1$, $k = 1,2,3,\ldots$, $\prod_{0\leqslant\tau_k<t}(1+c_k) < \infty$.
(H3) $d(\cdot)$ is a linear and continuous strictly monotonic increasing function of u satisfying $d(0) = 0$.

Here $(c_k), k = 1,2,3,\ldots$ are proportional coefficients. Impulsive reduction of the population is possible by catching or poisoning with chemicals used in agriculture $(-1 < c_k < 0)$, an impulsive increasing of the population is possible by artificial means by the population's impulsive immigration and introduction of natural enemies $(c_k \geqslant 0)$. In this paper, we assume $\tau_k = k\tau$ which means the individuals lay eggs only once all their life.

The dynamic of the delay system (1.1) and (1.2) has been studied in Stephen and Kuang (2009), we could obtain the positivity and boundedness of solution by ad hoc methods and global stability of the extinction and positive equilibria by the method of iteration. We also know that if the time adjusted instantaneous birth rate at the time of maturation is greater than 1, then the population will grow without being bounded. Thus, it is interesting how the dynamics of (1.3) is affected by the impulsive perturbations.

2. Preliminary

From Stephen and Kuang (2009), we could apply (1.2) recursively and could get a non-neutral delay equation

$$u'(t) = b_2^n e^{-ut}u_0((n+1)\tau - t) + b_1e^{-u\tau}\sum_{j=0}^{n-1} b_2^j e^{-j\mu t}u(t-(j+1)\tau) + d(u(t)) \quad t\in(n\tau,(n+1)\tau) \tag{2.1}$$

Since $\tau_k = k\tau$ so (1.3) can be written as

$$\begin{cases} u'(t) = b_2^n e^{-ut}u_0(\tau_{n+1} - t) + b_1e^{-u\tau}\sum_{j=0}^{n-1} b_2^j e^{-j\mu t}u(t-(\tau_{j+1}) \\ \qquad -d(u(t)) \quad t\in(n\tau,(n+1)\tau) \\ u(\tau_k^+) = (1+c_k)u(\tau_k) \quad k = 1,2,3,\ldots \end{cases} \tag{2.2}$$

Under the hypotheses (H1)–(H3), by a transformation $z(t) = \prod_{0\leqslant\tau_k<t}(1+c_k)^{-1}u(t)$, we consider the nonimpulsive delay differential equation

$$z'(t) = b_2^n e^{-ut}\prod_{\tau_{n+1}-t\leqslant\tau_k<t}(1+c_k)z_0(\tau_{n+1}-t) + b_1e^{-u\tau}\sum_{j=0}^{n-1}b_2^j e^{-j\mu t}\prod_{t-\tau_{j+1}-t\leqslant\tau_k<t}(1+c_k)^{-1}z(t-\tau_{j+1}) - d(z(t)) \tag{2.3}$$

and $t\in(n\tau,(n+1)\tau)$, $n = 1,2,3,\ldots$, with the initial condition

$$z_0(a) = z(0,a) = \prod_{0\leqslant\tau_k<t}(1+c_k)^{-1}u_0(a) \geqslant 0, \quad a \geqslant 0$$

Lemma 2.1. *Assume that (H1)–(H3) hold,*

(i) *If $z(t)$ is a solution of (2.3) on $(0,\infty)$, then $u(t) = \prod_{0\leqslant\tau_k<t}(1+c_k)z(t)$ is a solution of (2.2) on $(0,\infty)$.*
(ii) *If $u(t)$ is a solution of (2.2) on $(0,\infty)$, then $z(t) = \prod_{0\leqslant\tau_k<t}(1+c_k)^{-1}u(t)$ is a solution of (2.3) on $(0,\infty)$.*

Proof. First, we prove (i). It is easy to see that $z(t) = \prod_{0\leqslant\tau_k<t}(1+c_k)^{-1}u(t)$ is absolutely continuous on each interval (τ_k,τ_{k+1}), $k = 1,2,3,\ldots$, On the other hand, for every τ_k.

$$u(\tau_k^+) = \lim_{t\to\tau_k^+}\prod_{0\leqslant\tau_k<t}(1+c_j)z(t) = \prod_{0\leqslant\tau_k<t}(1+c_j)z(t) \text{ and}$$
$$u(\tau_k) = \prod_{0\leqslant\tau_k<t}(1+c_j)z(\tau_k)$$

Thus for every

$$k = 1,2,3,\ldots, \quad u(\tau_k^+) = (1+c_k)u(\tau_k) \tag{2.4}$$

Now, one can easily check that $u(t) = \prod_{0\leqslant\tau_k<t}(1+c_k)z(t)$ is a solution of (1.1) on $(0,\infty)$.

Next, we prove (ii), since $u(t)$ is absolutely continuous on each interval (τ_k,τ_{k+1}), $k = 1,2,3,\ldots$, and in view of (2.4), it follows that for any $k = 1,2,3,\ldots$.

$$z(\tau_k^+) = \prod_{0\leqslant\tau_k<t}(1+c_j)^{-1}u(\tau_k^+) = \prod_{0\leqslant\tau_k<t}(1+c_j)^{-1}u(\tau_k) = z(\tau_k)$$

and $z(\tau_k^-) = \prod_{0\leqslant\tau_k<t}(1+c_j)^{-1}u(\tau_k^-) = \prod_{0\leqslant\tau_k<t}(1+c_j)^{-1}u(\tau_k) = z(\tau_k)$. Which implies that $z(t)$ is continuous on $(0,\infty)$, it is easy to prove $z(t)$ is also absolutely continuous on $(0,\infty)$. Now one can easily check that $z(t) = \prod_{0\leqslant\tau_k<t}(1+c_k)^{-1}u(t)$ is a solution of (2.3) on $(0,\infty)$. The proof of Lemma 2.1 is complete.

3. Main results

Theorem 3.1. *Assume that (H1)–(H3) hold, and $z_0(a) \geqslant 0$ for all $a \geqslant 0$. Then the solution of (2.3) satisfies $z(t) \geqslant 0$ for all $t \geqslant 0$. Furthermore, if $z_0(a) = 0$ on the interval $(0,\infty)$, then $z(t) > 0$ for all $t > 0$.*