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Collision and intersection detection of two ruled surfaces using bracket method ☆

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ABSTRACT

Collision and intersection detection of surfaces is an important problem in computer graphics and robotic engineering. A key idea of our paper is to use the bracket method to derive the necessary and sufficient conditions for the collision of two ruled surfaces. Then the numerical intersection curve can be characterized. The cases for two bounded ruled surfaces are also discussed.

1. Introduction

Detecting collision of geometric objects has very important application in computer graphics, robotic engineering and computer animation. Some researchers have realized and studied the problem. Boyse (1979) discussed the detection of intersections among objects in fixed positions and collisions among objects moving along specified trajectories. Moore and Wilhelms (1988) divided the problem to a kinematic problem and a dynamic one. An algorithm for detecting intersection between n spheres was presented in Hopcroft et al. (1983). For high precision, the determining conditions for the relationship of geometric models are studied. For an instance, Wang et al. (2001) gave an algebraic condition for the separation of two ellipsoids.

The bracket can be defined as an algebraic tool to represent projective invariants symbolically (Hodge and Kromann, 1953). In this paper, we give the necessary and sufficient conditions for positional relationship of two space lines and two space line segments by bracket method. The representation with brackets offers a simple description for geometric relationship. That is to say, we can directly judge the intersection or the separation according the symbolic formula given in this paper. An advantage of our method is avoiding the redundant discussion and increasing the performance efficiency.

Rational ruled surfaces are an important class of algebraic surfaces which is widely used in computer aided geometry design. According to Chen (2003) and Li et al. (2008), people can find a simplified parametrization from a given ruled surface. In further considerations, it is necessary to determine the geometric relationship of two surfaces, including collision detection and intersection curve analysis. Heo et al. (1999) discussed the intersection of two parametric ruled surfaces, their idea is straightforward but the computation is a little complicated. Fioravanti et al. (2006) gave a way to compute the

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intersection by implicitizing one ruled surface and the topology analysis of a planar curve. The implicitization of the surface was based on the resultant and greatest common division (GCD) computations.

As an application of the bracket method, this paper discusses the positional relationship of two ruled surfaces. Based on some results on lines and line segments, we present the conditions for the collision of two ruled surfaces. We also give a characterization of intersection expressions with two parameters. Some conditions we obtain are similar to that in Heo et al. (1999), since we focus on the same problem. However, we simplify the analysis by dividing the intersection into two parts: overlapping rule lines and ordinary intersection points. And we need not to consider the degenerate cases which were discussed in Heo et al. (1999). The degree of the intersection expression is also decreased by setting certain specialized auxiliary points. Once the expression of the intersection is given, we can compute the numerical intersection using the typical process in Fioravanti et al. (2006) and we take a new algorithm for planar curves topology analysis from Cheng et al. (2009). But we do not need the implicitization process based on resultant and GCD computations as in Fioravanti et al. (2006). Furthermore, we discuss the conditions for collision of two ruled surface segments. This situation has more practical applications and the methods in Heo et al. (1999), Fioravanti et al. (2006) cannot be generalized to cover this problem easily. We reduce the collision detection to solving real solutions of a semi-algebraic system (SAS). If the time parameter is included, we can find the time interval of collision by solving SAS and quantifier elimination.

The rest of this paper is organized as follows. In Section 2, some notations and preliminaries are introduced. In Section 3, we give the conditions for characterizing positional relationship of two lines and line segments. In Section 4, we discuss the conditions for the intersection of two ruled surfaces and compute the numerical intersection. In Section 5, collision detection of two ruled surface segments are discussed. In Section 6, we summarize the paper.

2. Preliminaries

In this section, we introduce the notations needed in our discussion. Let $\mathbb{R}[u]$ be the ring of polynomials in u over the field of real numbers, and $\mathbb{R}[u]^4$ the set of column vectors of size four whose entries belong to $\mathbb{R}[u]$. A *rational ruled surface* is defined as a bi-degree $(n,1)$ tensor product rational surface:

$$(x,y,z)^T = \mathbf{P}(u,s) = \mathbf{P}_1(u)(1-s) + \mathbf{P}_2(u)s, \tag{2.1}$$

where $\mathbf{P}_i(u), i = 1,2,$ are rational curves, called the *directrices* of $\mathbf{P}(u,s)$, and $\mathbf{P}_1 \neq \mathbf{P}_2$. We assume that the rational parametrization (2.1) is nontrivial, that is, it defines a surface $f(x,y,z) = 0$.

For a fixed $u = u_0$, $\mathbf{P}(u_0,s) = \mathbf{P}_1(u_0)(1-s) + \mathbf{P}_2(u_0)s$ is a ruling line of the ruled surface. If another ruled surface $\mathbf{Q}(v,t) = \mathbf{Q}_1(v)(1-t) + \mathbf{Q}_2(v)t$ in the form of (2.1) intersects with $\mathbf{P}(u,s)$, then there exist u_0 and v_0 such that two ruling lines $\mathbf{P}(u_0,s)$ and $\mathbf{Q}(v_0,t)$ are intersected. Then the problem of computing intersection of two ruled surfaces is reduced to that of two moving lines.

We will apply the bracket method to analyze the intersection of lines effectively.

Definition 1. In $\mathbb{R}^n$, $n+1$ points $\mathbf{x}_1,\dots,\mathbf{x}_{n+1}$ with the coordinates form $\mathbf{x}_i = (x_{i1},\dots,x_{in})$ for $i = 1,\dots,n+1$, the bracket $[\mathbf{x}_1 \dots \mathbf{x}_{n+1}]$ is defined as follows:

$$[\mathbf{x}_1 \dots \mathbf{x}_{n+1}] = \begin{vmatrix} x_{11} & \dots & x_{(n+1)1} \\ \vdots & \ddots & \vdots \\ x_{1n} & \dots & x_{(n+1)n} \\ 1 & \dots & 1 \end{vmatrix}.$$

If the number of the points is $m+1 < n+1$, then they will determine a hyperplane with dimension less than $m+1$. In this situation, we will define the bracket of these points in a hyperplane of dimension m.

Definition 2. For a hyperplane $H \subset \mathbb{R}^n$, let its dimension be m, where $m < n$. Then there exist an $n \times n$ orthogonal matrix O_H and a vector $t_H \in \mathbb{R}^n$, for any $m+1$ points $\mathbf{x}_1,\dots,\mathbf{x}_{m+1} \in H$ with the coordinates form $\mathbf{x}_i = (x_{i1},\dots,x_{in})^T$ where $i = 1,\dots,m+1$, such that

$$O_H \begin{pmatrix} x_{11} & \dots & x_{(m+1)1} \\ \vdots & \ddots & \vdots \\ x_{1n} & \dots & x_{(m+1)n} \end{pmatrix} + (t_H,\dots,t_H) = \begin{pmatrix} x'_{11} & \dots & x'_{(m+1)1} \\ \vdots & \ddots & \vdots \\ x'_{1m} & \dots & x'_{(m+1)m} \\ 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{pmatrix}.$$