Computer-Aided Design 78 (2016) 147–157

Contents lists available at ScienceDirect

Computer-Aided Design

journal homepage: www.elsevier.com/locate/cad

A closed-form formulation of HRBF-based surface reconstruction by approximate solution

Shengjun Liu [a,b], Charlie C.L. Wang [c,*], Guido Brunnett [d], Jun Wang [e]

[a] State Key Laboratory of High Performance Manufacturing Complex, Central South University, Changsha, Hunan 410083, China
[b] Institute of Engineering Modeling and Scientific Computing, Central South University, Changsha, Hunan 410083, China
[c] Department of Design Engineering, Delft University of Technology, The Netherlands
[d] Department of Computer Science, Chemnitz University of Technology, Chemnitz 09126, Germany
[e] College of Mechanical and Electrical Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

ARTICLE INFO

Keywords:
Hermite Radial Basis Functions
Quasi-solution
Closed-form
Surface reconstruction

ABSTRACT

The *Hermite radial basis functions* (HRBFs) implicits have been used to reconstruct surfaces from scattered Hermite data points. In this work, we propose a closed-form formulation to construct HRBF-based implicits by a quasi-solution to approximate the exact one. A scheme is developed to automatically adjust the support sizes of basis functions to hold the error bound of a quasi-solution. Our method can generate an implicit function from positions and normals of scattered points without taking any global operation. Robust and efficient reconstructions are observed in our experimental tests on real data captured from a variety of scenes.

1. Introduction

Reconstructing surface from a set of unorganized points equipped with normal vectors is an important topic in various fields such as computer graphics, reverse engineering, image processing, mathematics, robotics, computer-aided design and manufacturing. A lot of research approaches have been devoted to developing surface reconstruction methods, in which implicit surface fitting based on *Radial Basis Functions* (RBF) is successful in dealing with noisy and incomplete data (e.g., [1–3]).

Recently, implicits based on *Hermite Radial Basis Functions* (HRBF) were presented to interpolate data points to the first order in [4]. It is robust and effective to deal with coarse and non-uniformly sampled points, close surface sheets, and able to produce surface reconstruction with details. However, interpolating both positions and normals of points leads to the computation of solving a $4n \times 4n$ linear system for an input with n points. It is impractical due to the expensive computation. The system becomes sparse when the *Compactly Supported Radial Basis Functions* (CSRBF) are used as the kernel functions. However, this attempt on improving the efficiency can also bring in a more challenging problem of numerical stability. A closed-form formulation is presented in this paper to solve the computational problem of HRBF-based reconstruction by quasi-interpolation. Our formulation results in a kind of approximate interpolants that fit implicits by weighted averages of the values at given points. The most attractive property of this approach is able to robustly construct a surface from a set of points without solving linear systems—i.e., with a closed-form formulation (see Section 3.1). This can make the computation of HRBF-based surface reconstruction stable and efficient. As shown in Fig. 1, a mesh surface with 331k triangles can be efficiently reconstructed from an input set with 922k points by our method in 5.5 s. Comparing to the recent *Floating Scale Surface Reconstruction* (FSSR) [5] that also avoids applying global operations, our method is more than 17.9× faster. Moreover, we have analyzed the error-bound between our closed-form solution and the solution obtained by solving linear systems (see Section 3.2). Specifically, the error-bound exists when the number of points covered by every support of all CSRBFs is capped by a fixed number, m. Experimental tests have been conducted to verify the efficiency and robustness of our approach.

1.1. Contributions

In this paper, we propose a closed-form formulation for computing the approximate solution of HRBF-based surface reconstruction from scattered data points.

* Corresponding author. Tel.: +31 1 527 88406.
E-mail address: c.c.wang@tudelft.nl (C.C.L. Wang).

http://dx.doi.org/10.1016/j.cad.2016.05.001
0010-4485/

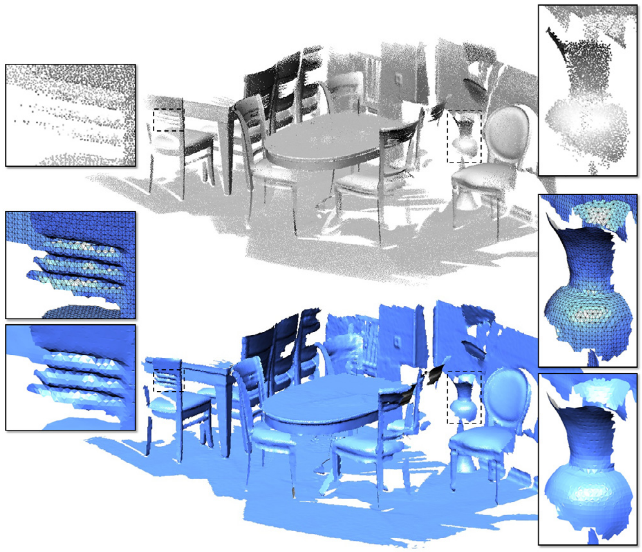

Fig. 1. The method proposed in this paper can efficiently reconstruct a surface from a set of noisy and incomplete points—e.g., the indoor scene shown here with 922k points. Our reconstruction takes only 5.5 s to generate a mesh surface with 313k triangles have the similar quality as the state-of-the-art [5] but our approach is 17.9× faster. Note that the scale factor used in FSSR [5] is generated as 3× of support size used in our approach so that similar number of triangles are generated, and other default parameters are used for FSSR.

- Our method can construct a signed scalar function by directly blending the positions and normals of points without any global operation. The computation based on compact support is local and numerically stable.
- Errors between the quasi-solution and the exact one are bounded when applying an automatic scheme to adjust the support sizes of basis functions.
- Our formulation to find the approximate solution of HRBF-based surface reconstruction is robust. When compared with other approaches, this method can still successfully reconstruct a satisfactory surface on highly noisy points with up to 60% Gaussian noise.
- As a local approach, our method is efficient and scalable. This is well-suited for highly parallel implementation as well as distributed/progressive reconstruction.

Note that, the compactly-supported basis functions result in open meshes and leave holes if the region does not have enough number of points, which actually fits the application of reconstructing outdoor scenes very well. Such holes also provide cues for the reliability of a reconstruction that can be used to supervise the active scanning and reconstruction (Ref. [6]).

1.2. Related work

The problem of surface reconstruction from point cloud has been studied in literature for more than two decades. After using signed distance field in [7] to reconstruct mesh surface from point clouds, implicit functions have gained a lot of attention in surface reconstruction because of its ability to handle topological change and filling holes. Example approaches include RBF-based methods [4,1,2,8–16], Poisson surface reconstructions [17,18], smooth signed distance method [19], and *Partition-Of-Unity* (POU) based methods [20,21]. A comprehensive review of all these works has beyond the scope of this paper. More discussion and comparison on different surface reconstruction methods can be found in [22] and the recent survey paper [23]. Here we only review methods that are closely related to our approach.

The methods based on RBF implicits are popular for their capability of handling sparse point clouds. Generally, RBF-based methods transform the reconstruction into a multi-variable optimization problem, where enforcing the interpolation constraints results in a linear system. Solving the linear system is an important but time-consuming step for the RBF-based reconstruction. To obtain a non-trivial solution, RBF-based methods usually require the provision of extra offset-points (Refs. [1,2]) that can be obtained by shifting data points along their normal directions. However, it is not easy to find an optimal offset distance. The positions of these offset points is also difficult to determine (especially when the scanned model has small features). To avoid generating offset-points, Ohtake et al. [9,13] used a signed function which includes basic approximations and local details. Some prior works [10,15], deduced from the statistical-learning perspective, avoid generating offset-points in surface reconstruction, where normals were directly used in a variational formulation. Recently, Macedo et al. [4] derive an implicit function from the Hermite–Birkhoff interpolation with RBFs. They enhance the flexibility of HRBF reconstruction by ensuring well-posedness of an interesting approach combining regularization. However, given a set with n Hermite points, these methods result in a $4n \times 4n$ linear system to be solved, which limits the number of points in reconstruction.

Quasi-interpolation is a simple, efficient, and computational stable method in the field of function approximation. In the early work of quasi-interpolation [24], a function approximating a given data set is defined by a weighted average of the values at the data points. The quasi-interpolation with RBF kernels has been studied in [25,26], which is later employed for surface reconstruction [27,28]. However, when the problem extended to HRBF based surface reconstruction, these quasi-interpolation techniques cannot be directly applied as the implicit function of HRBF does not follow the required form of quasi-interpolation. Our work presented in this paper overcome this difficulty. To the best of our knowledge, this is the first approach that provides a closed-form solution for HRBF-based surface reconstruction.

Generally speaking, *Moving Least-Squares* (MLS) based methods [29–34], also construct a semi-implicit function by weighted sum of data points. Normal constraints are enforced by either approximation or interpolation in a few MLS approaches (Refs. [32,30,34]). The method of Shen et al. [32] is mainly for polygonal meshes. Therefore, we only compare our results with *Algebraic Point Set Surfaces* (APSS) [30] and *Hermite Point Set Surfaces* [34] in the experimental tests and find that our method is more robust on noisy input. In the recent work named as FSSR [5], weighted average is also employed to fit implicit functions to the input set of points. The final implicit functions are formed by locally supported basis functions satisfying the property of POU. However, FSSR is 8.5× to 36.4× slower than our method when generating a reconstruction with similar number of triangles. More details can be found in Section 5.

2. HRBF implicits

The HRBF implicits [4] are built upon the theory of Hermite–Birkhoff interpolation with radial basis functions [3]. In this section, we briefly describe how to use HRBF implicits to solve the problem of surface reconstruction from scattered points.

Definition 1. Given a set of data $\mathcal{P} = \{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n\}$ with unit normals $\mathcal{N} = \{\mathbf{n}_1, \mathbf{n}_2, \ldots, \mathbf{n}_n\}$, the HRBF implicits give a function f interpolating both the points and the normal vectors as

$$f(\mathbf{x}) = \sum_{j=1}^{n} \{a_j \varphi(\mathbf{x} - \mathbf{p}_j) - \langle \mathbf{b}_j, \nabla \varphi(\mathbf{x} - \mathbf{p}_j) \rangle\}, \tag{1}$$