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A new approach to automatic and a priori mesh adaptation around circular holes for finite element analysis

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ABSTRACT

Through our research on the integration of finite element analysis in the design and manufacturing process with CAD, we have proposed the concept of mesh pre-optimization. This concept consists in converting shape and analysis information in a size map (a mesh sizing function) with respect to various adaptation criteria (refining the mesh around geometric form features, minimizing the geometric discretization error, boundary conditions, etc.). This size map then represents a constraint that has to be respected by automatic mesh generation procedures. This paper introduces a new approach to automatic mesh adaptation around circular holes. This tool aims at optimizing, before any FEA, the mesh of a CAD model around circular holes. This approach, referred to as "a priori" mesh adaptation, should not be regarded as an alternative to adaptive a posteriori mesh refinement but as an efficient way to obtain reasonably accurate FEA results before a posteriori adaptation, which is particularly interesting when evaluating design scenarios. The approach is based on performing many offline FEA analyses on a reference case and deriving, from results and error distributions obtained, a relationship between mesh size and FEA error. This relationship can then be extended to target user specified FEA accuracy objectives in a priori mesh adaptation for any distribution of circular holes. The approach being purely heuristic, fulfilling FEA accuracy objectives, in all cases, cannot be theoretically guaranteed. However, results obtained using varying hole diameters and distributions in 2D show that this heuristic approach is reliable and useful. Preliminary results also show that extension of the method can be foreseen towards a priori mesh adaptation in 3D and mesh adaptation around other types of 2D features.

1. Introduction

The rapid evolution of computer systems and computer aided design (CAD) systems allows for significant increase in performance when solving engineering problems with numerical methods. Amongst these methods, finite element analysis (FEA) [1] has undergone a major expansion for the last twenty years, despite the development of several alternative methods (meshless methods like the Element Free Galerkin Method [2], isogeometric analysis [3] and XFEM methods [4] for example) that have proven efficiency in the analysis of certain classes of engineering problems. Only operating on large size computer systems thirty years ago, FEA can now be processed on personal computers and represents an affordable and versatile tool for solving problems in the context of computer aided product design. Even if its successful use still requires significant expertise, availability of FEA has increased and will certainly further increase in the future. This democratization in the use of numerical analysis induces that it is now used by various actors through the product design cycle (from high-end analysts to early stages designers) which also induces increasingly sophisticated requests on behalf of users. To assist designers and analysts, many tools have been developed towards reducing time required to derive FEA models from CAD product models and towards making its use easier for less experienced actors. Integrating FEA with in the CAD world is one of the early concepts [5,6] that first came out to make FEA more accessible and productive. This integration between CAD and FEA is now a reality in many commercial solutions even if work remains to be done towards achieving this integration into the product design cycle itself.

Moreover, during the last fifteen years, accessibility and productivity of FEA has also increased through a lot of work performed with regard to CAD/FEA integration [7–9], to automatic mesh generation [10–12] and to mesh adaptation based on error

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estimation [13]. These tools have dramatically lowered time required to obtain accurate FEA results and have also allowed less experienced FEA users, like early stage designers, to get their designs verified with a certain level of confidence about the accuracy of their analysis results. More generally, it appears that a major consequence of these advances is the fact that FEA technology is now used by a much wider number of actors along the design and manufacturing process, which induces a need for tools allowing less skilled users to obtain reasonably accurate FEA results. Building this type of tools is not simple since it fundamentally involves knowledge synthesis about the way good FEA models can be derived from CAD and about the way relevant engineering conclusions can be drawn from raw FEA results.

In traditional approaches to mesh adaptation, a coarse initial mesh is gradually refined through several analysis and error estimation loops, which provides, at the end, analysis results with a controlled level of accuracy. In this context, several authors, among which Zhu and Zienkiewicz [14] and Kang and Haghighi [15,16], showed that the attributes (element size distribution and quality) of the initial mesh introduced as input of this iterative process are crucial to obtain accurate results, with a reasonable number of iterations and without using a very large number of degrees of freedom (DOF). Consequently, it appears that introducing, at the beginning of this iterative process, a mesh with reasonably refined element sizes in specific zones, results in faster convergence with less DOF. The problem is that this is clearly out of reach for certain types of FEA users. On the other hand, more experienced FE analysts use their expertise to build this type of initial meshes, which means featuring *a priori* adaptation in sensitive zones. Note that, all along the paper, *a priori* means before any FEA. *A priori* is opposed to *a posteriori*, which refers to the automatic refinement process based on FEA error estimation as described above.

Also, since FEA is used through the entire design cycle, differences must be made with regard to accuracy requirements between early design, detailed design and design optimization activities. Usually, during early design steps, analysts only require a reasonable level of accuracy, which means that an initial mesh with an *a priori* reasonable adaptation in sensible zones would often be sufficient to meet accuracy requirements (see Fig. 1). Here again, this is out of reach for less experienced users and, for more experienced users, even if is not technically complex, it is usually long and fastidious. In this context, productivity in using FEA at various stages of the design process could notably be increased with the use of efficient and automatic *a priori* mesh adaptation tools. Even if, as presented in the next section, several approaches to *a priori* automatic mesh adaptation have been proposed in the literature, setting up robust and efficient tools in this direction still represents a major challenge for the following reasons:

- *A priori* mesh adaptation is intimately related to the experience and knowledge of engineering problems considered. FEA is used for solving numerous and various engineering problems such as elasticity, heat transfer and magnetics, in stationary and transient state and for both linear and non-linear problems. Consequently, even if similar adaptation rules can be applied in the case of distinct engineering problems, *a priori* automatic adaptation may be different in different analysis contexts.
- *A priori* mesh adaptation fundamentally relies on knowledge synthesis, and the knowledge on which *a priori* adaptation fundamentally relies is usually very complex.
- The knowledge to be synthesized applies on various types of input data (geometric features, material behavior, boundary and loading conditions, etc.) and this data can be vague, ambiguous, extremely delicate to identify and synthesize.
- The process has to be completely automated to be practically relevant and efficient, which means automating both data identification, decision making and decision applying.

Fig. 1 illustrates the flowchart of our integrated CAD/FEA environment [17,18] and situates automatic *a priori* mesh adaptation, in the process, if compared to *a posteriori* mesh adaptation. In this flowchart, the CAD model (Fig. 1(a)) is first meshed using *a priori* mesh adaptation (Fig. 1(b)), which induces a first FEA result (Fig. 1(c)). The "a priori" adapted mesh, introduced at the beginning of the process, is then eventually refined through several loops of "a posteriori" mesh refinement (Fig. 1(e)), which is based on classical FEA error estimation procedures (Fig. 1(d)). This flowchart also illustrates that, in some cases, *a posteriori* mesh adaptation is not necessary and that, in these cases, a good *a priori* mesh adaptation is sufficient. In fact, as mentioned just above, it strongly depends on the context and objectives of analysis. Moreover, Fig. 1 also illustrates that *a posteriori* mesh adaptation refines the mesh in zones where the mesh obviously does not need to be refined (around imposed displacement in this case). Since *a posteriori* mesh adaptation is only based on FEA error estimation, and not on knowledge synthesis, its major drawback is that it may refine the mesh in zones where it is not relevant. However, it is important to mention that *a priori* mesh adaptation is not intended at replacing *a posteriori* mesh adaptation. It has to be seen as an additional tool that can be used towards performing accurate FEA simulations easier and faster.

It is important to outline, as described in [19,12,20], that CAD, FEA, size and mesh data are closely integrated together in CAD/FEA integrated platforms. This allows rapid changes to design solutions along the design process, while keeping track of the engineering knowledge accumulated through the evolution of these solutions.

Fig. 1 also points out that mesh adaptation is practically based on setting up size maps. A size map (or sizing function) is formulated as a 3D scalar field $E(x, y, z)$ in the case of isotropic mesh grading. Even if this is not used in the work presented in this paper, it is worth noting that mesh grading, either isotropic or anisotropic, is usually represented using a *metric* [11], which is defined as matrix field $M(x, y, z)$.

The main objective of this paper is introducing a new methodology towards implementing and automating *a priori mesh adaptation* in the context of **2D and 3D linear elasticity problems**. At this point, its implementation is limited to mesh adaptation around circular holes but its extension to other types of features can reasonably be foreseen. This paper is organized as follows. In Section 2, we present existing approaches to automatic *a priori* mesh adaptation and their limitations, with a specific focus on a previous work led by our research team on the subject [17]. It ends with the objective of our research. Section 3 details the approach proposed towards automatic and *a priori* mesh adaptation around circular holes and presents validations for different holes distributions. The way a size map is represented and saved is explained in Section 4, followed by validation examples in 3D in Section 5. The paper ends with comments related to extending the approach to other types of features in Section 6 and with a conclusion about perspectives of future work.

2. A priori mesh adaptation: literature review and objectives

2.1. Introduction

We started working on a priori mesh adaptation several years ago [5,17,18] and, in our work, it is referred to as *nodal density pre-optimization*. Several approaches to a priori mesh adaptation have been proposed, with varying generality and varying level of automation. In this literature review, we will focus on the following most important issues in the process and classify references with respect to these issues:

- Knowledge synthesis and decision making.
- Criteria underlying a priori mesh adaptation.
- Deriving size maps from data extraction and knowledge synthesis.