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Cell packing structures

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HIGHLIGHTS

- Recent and ongoing research in architectural geometry.
- Links between cell packing structures and discrete differential geometry.
- Applications, e.g. to shading and indirect lighting.
- Interplay of geometry, optimization, statics, manufacturing.
- Combining form, function and fabrication into novel design tools.

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ABSTRACT

This paper is an overview of architectural structures which are either composed of polyhedral cells or closely related to them. We introduce the concept of a support structure of such a polyhedral cell packing. It is formed by planar quads and obtained by connecting corresponding vertices in two combinatorially equivalent meshes whose corresponding edges are coplanar and thus determine planar quads. Since corresponding triangle meshes only yield trivial structures, we focus on support structures associated with quad meshes or hex-dominant meshes. For the quadrilateral case, we provide a short survey of recent research which reveals beautiful relations to discrete differential geometry. Those are essential for successfully initializing numerical optimization schemes for the computation of quad-based support structures. Hex-dominant structures may be designed via Voronoi tessellations, power diagrams, sphere packings and various extensions of these concepts. Apart from the obvious application as load-bearing structures, we illustrate here a new application to shading and indirect lighting. On a higher level, our work emphasizes the interplay between geometry, optimization, statics, and manufacturing, with the overall aim of combining form, function and fabrication into novel integrated design tools.

1. Introduction

The formation of complex materials and structures from smaller and simpler cells is an omnipresent concept in nature and certainly also in architecture. While nature is abundant in shapes of cells, architecture – at least so far – had to be more conservative and uses simple cells which can be easily manufactured. Hence, we here deal mostly with polyhedral cells, i.e., those which exhibit flat faces. Since part of contemporary architecture favors organic freeform shapes, we will discuss the following basic question: *How can one generate freeform shells by tightly packing polyhedral cells*? We mainly focus on just one layer of cells, although we will comment on how to generate multiple layers. In the actual architectural realization, the polyhedral cells may not be completely realized or may be decorated with additional detail (see Figs. 1 and 4). Moreover, we discuss derived structures which are not really polyhedral anymore, but can easily be generated from polyhedral cell packings.

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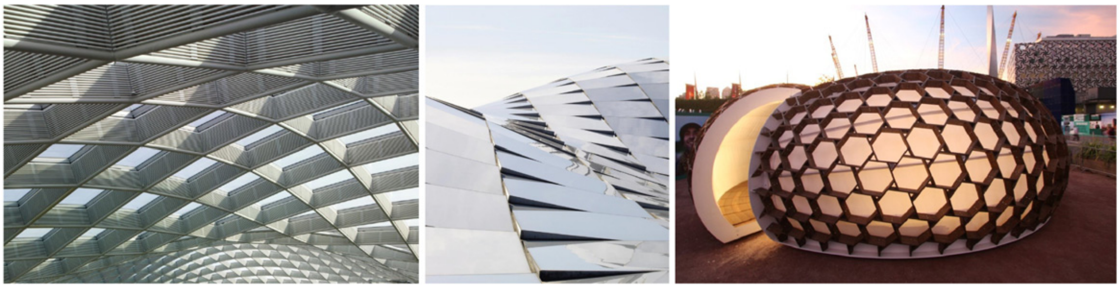

Fig. 1. Real projects exhibiting cell packing structures: *left and center* – details from the roof of the Kogod Courtyard, Smithsonian National Portrait Gallery, Washington DC, by Foster + Partners; *right* – KREOD Pavilions, by Pavilion Architecture, London (Architect: Chun Qing Li).

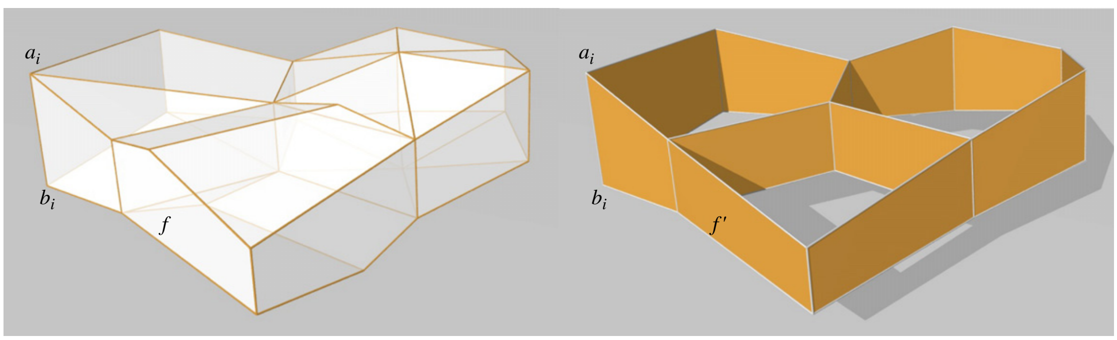

Fig. 2. *Left*: cell packing of three polyhedra. *Right*: the support structure of this polyhedral cell packing. It consists of quadrilaterals f' which are spanned by the inner edges of inner faces f of the packing. We indicate one of the inner edges a_ib_i; it spans a so-called node axis of this support structure.

The present paper may be seen as a survey with new results. It covers some core developments in Architectural Geometry of the past few years, but is written from a somewhat different perspective. We therefore do not discuss prior research at the beginning of our paper, but prefer to integrate it into the relevant sections.

Our paper is organized as follows: In Section 2, we formulate the problem, introduce the basic terminology and concepts and focus on cell arrangements related to Voronoi diagrams and hexagonal patterns. Section 3 presents a survey of packings with quadrilateral combinatorics and its close relations to discrete differential geometry. Aspects of statics, especially self-supporting surfaces, and the rigidity of the presented structures are discussed in Section 5. Throughout the paper, we illustrate the discussions with various applications. Section 6 concludes the paper.

The Material Ecology aspect. Material Ecology is an interdisciplinary research initiative which, we quote, "undertakes design research in the intersection between architecture, engineering, computation, and ecology [… It] undertakes research in advanced digital applications for architectural practice and pursuits their contribution to a design paradigm promoting generative design processes". The study of geometric and physical aspects of freeform architecture certainly fits most of these items directly or indirectly, in particular it lies in the intersection between computation, architecture and engineering. Our philosophy could be described as analytic and knowledge-based, laying the foundations for generative design. Guiding thoughts are efficiency of construction, and also the exploration of the most general shapes obtainable from specified local geometric and other constraints. Since often local geometry (cell shapes) is inspired by forms occurring in nature – most famously in honeycombs or radiolaria – we might argue that we are borrowing shapes from a source which already has performed specific optimization on them. This again puts our research in the context of Material Ecology.

2. Polyhedral cell structures

2.1. Basic types and properties of polyhedral cell structures

A polyhedron is a solid whose boundary has planar faces only; if no confusion can arise, also the boundary surface is called a polyhedron. The polyhedra shall be tightly packed in order to form a thick shell (see Fig. 2). Shared faces of neighboring polyhedra are called *inner faces* or *transverse faces*; the other faces are *boundary faces*. Analogously, a polyhedral packing has *inner edges* and *boundary edges*.

The support structure of a polyhedral cell packing. Packing polyhedra in this way, the planes of the inner faces, confined by the lines of the inner edges, form a structure which is of central importance for our considerations. As we want the packing to have nonzero thickness everywhere, each inner face contains two inner edges, which we can connect to form a quad (see Fig. 2). The resulting arrangement of quads will be called the *support structure* of the polyhedral cell packing. This name also indicates an important application: One can align prismatic beams of a load-bearing structure with its faces (see e.g. Fig. 1, left). In this case we speak of a *torsion-free support structure*, and the inner edges are called the *node axes*.

Polyhedral plate structures. In some cases it is possible to close all cells of the support structure with just two flat faces, one on either side. Then we have two combinatorially equivalent polyhedral meshes A, B, which form the boundary of a structure we refer to as a *polyhedral plate structure*. It is obtained by joining corresponding vertices and edges in meshes A and B (which have the particular property that corresponding edges are coplanar). One may see meshes A and B as variable distance offsets of each other.

Parallel meshes and offsets. A practically useful special case occurs when corresponding faces of A and B lie in parallel planes; then also corresponding edges are parallel. Such *offset meshes* have been investigated in detail [1]. If the plates are of constant thickness