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# $G^2$ quasi-developable Bezier surface interpolation of two space curves☆

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## HIGHLIGHTS

- We model a quasi-developable surface interpolating two arbitrary space curves.
- The surface is represented by an aggregate of four-sided Bezier patches.
- These patches can be optimally assembled in terms of developability degree.
- The resultant surface can obtain $G^2$ continuity.

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## ABSTRACT

Surface development is used in many manufacturing planning operations, e.g., for garments, ships and automobiles. However, most freeform surfaces used in design are not developable, and therefore the developed patterns are not isometric to the original design surface. In some domains, the CAD model is created by interpolating two given space curves. In this paper, we propose a method to obtain a $G^2$ quasi-developable Bezier surface interpolating two arbitrary space curves. The given curves are first split into a number of piecewise Bezier curves and elemental Bezier patches each of which passes through four splitting points are constructed. All neighboring elemental patches are $G^2$ connected and they are assembled optimally in terms of the degree of developability (the integral Gaussian curvature). Experiments show that the final composite Bezier surface is superior to a lofted one which is defined regardless of the final surface developability.



## 1. Introduction

Surface developability is highly demanded for manufacturing considerations in industrial applications like garment design [1], sheet-metal forming [2–4] and windshield design [5], in which products are often made from planar patterns by welding, pumping or sewing. In these industries, designers often sketch out two space curves, and hope to interpolate them with a developable or nearly developable surface. In practice, the designer uses a loft or skinning method to build a NURBS surface passing through the given curves, and then analyzes the resultant surface to see whether it can be flattened into one or several planar patterns within the allowable distortion. If not, the curves will be repeatedly modified until the desired developability of the loft surface can be obtained. Lofting or skinning is only a general interpolating method which disregards the developability geometric constraint; they suffer from among others one particular problem: the curves may have been modified too much to meet the developability constraint that their final shapes no longer represent the designer's original intent. Furthermore, if curves themselves are complicated, using the above method to model a developable surface will often become impossible.

In this paper, we propose a novel method to construct a $G^2$ composite of four-sided Bezier patches to interpolate two arbitrary space curves. These patches are optimally assembled together in terms of developability. The algorithm is inspired by the work of [6]. The work of [6] uses planar triangles to interpolate two polylines and an optimal triangulation can be obtained from the given polylines. Our work extends it to polynomial surfaces, and each pair of the neighboring patches will be $G^2$ connected, while in [6] planar triangles are only $G^0$ connected (see Fig. 1). $G^2$ continuity is sometimes necessary for the purpose of both aesthetic and functional requirement in engineering. For example, in CAM design, $G^2$ can avoid creating abrupt changes on a surface; in sheet metal applications such as a ship hull and the wing and body of an aircraft, $G^2$ can prevent flow separation and turbulence. In parametric developable surface design, due to high complexity,

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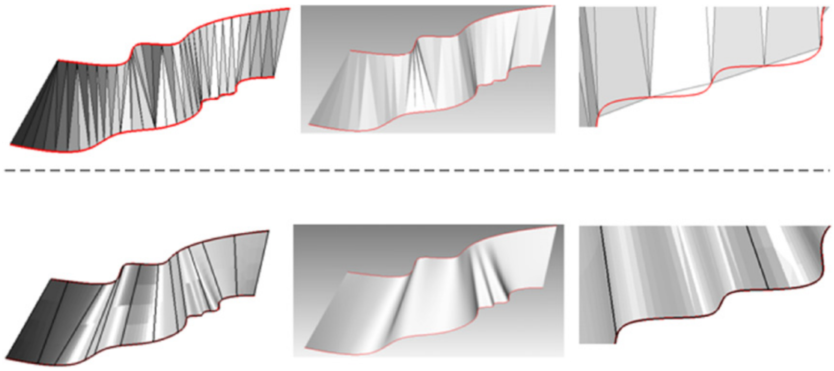

**Fig. 1.** Our method vs. [6]. Top (the method of [6]): (from left to right) the optimal triangulation, shade view ($G^0$ obtained), interpolation error due to linear approximation (the red curve is the original curve); bottom (our method): (from left to right) the optimal composite surface made of Bezier patches, shade view ($G^2$ pleasing visual appearance), perfect interpolation at the boundary. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

most works deal with only $G^1$ continuity, while our objective is to design a $G^2$ composite Bezier surface to interpolate two given space curves.

In our work, as the tile elements are Bezier patches instead of planar triangles, we do not require the distance between the two given curves to be much shorter than the lengths of the given curves [6]. In addition, the $G^2$ continuity rather than $G^0$ in [7,6,8] and $G^1$ in [9] bring along pleasing visual results preferred in many industrial applications. Our entire problem can be decomposed into two-sub ones: (1) how to construct a $G^2$ Bezier patch with only the local information of its four corner points, as in the construction process the exact parametric form of its neighboring elemental patch is yet to be decided; and (2) how to modify the algorithm in [6] (which uses flat triangles only) to accord to a four-sided Bezier patch assembly.

The rest of the paper is organized as follows. Section 2 reviews the related works. Section 3 defines the problem, with the problem formulation and a general description of our proposed algorithm. Sections 4 and 5 provide details of our methodology, while Section 6 shows the experimental results with discussions. Section 7 concludes the paper.

## 2. Related work

There is a rich body of study in developable surfaces, especially in the context of NURBS or *B*-Spline surfaces [10–16]. In [12] a condition is proposed under which a developable Bezier surface can be constructed with two boundary curves. The boundary curves in [12] are of at most degree 3 and are restricted to lie in parallel planes. Their work was extended by Frey and Bindschadler [14] by generalizing the degree of the directrices. Maekawa and Chalfant [16] extended Aumann's algorithm to *B*-Spline curves by segmenting the original *B*-Spline curves into multi-segment planar Bezier curves. Chu and Sequin [13] proposed a new method to design a developable Bezier patch. In their method, after one boundary curve is freely specified, five more degrees of freedom are available for the second boundary curve of the same degree. Aumann [10,11] utilized De Casteljau algorithm to design developable Bezier surfaces through a Bezier curve of arbitrary degree and shape. However, the approach often results in unexpected or undesirable surfaces. In the work of [4], a quasi-developable *B*-spline surface of degree $1 \times n$ can be determined after searching through a series of ruling pairs. Projective geometry methods exploiting point–plane duality were investigated by the groups of Ravani and Pottmann [17–21].

Some researchers have used spatial kinematics and line geometry to approximate a given surface by a developable one [22–24]. The idea is to generate a developable surface by sweeping a line along a helical motion. This method is widely used in reverse engineering. Given a set of scattered data points, it is required to find a developable surface of which the generator points are as close as possible to the given points. In Hoschek and Schneider [23,24], the distance from a point to a line was used, leading to a nonlinear optimization problem. Pottmann and Wallner [21] and Hoschek and Schwanecke [23,24] independently introduced the error measurement between planes, leading to linear algorithms. Special attention was paid to controlling the regression curve and how to combine the pieces of patches with imposed continuity conditions.

Recently, methods based on discrete data representation have been proposed for design of quasi-developable surfaces, owing to the rapid increase of computing power and popularity of 3D meshes. In this realm, triangle or quad meshes are sought after to achieve maximum developability, with the given interpolating constraints satisfied. In his pioneering work, Frey [25] showed how to approximate buckled binder wrap surfaces by calculating out the *d*-vertices based on the fact that Gaussian curvature at every point on a developable surface is zero, and this condition can be expressed by requiring the internal angle at every internal vertex to be $2\pi$. In [26], triangular strips are designed by grouping original triangles on the mesh which share similar topological distances, and the resulting pattern gives out a near-developable unfolding of the mesh. Wang and Tang [27] minimized the total discrete Gaussian curvature for a polygonal surface by relocating each mesh vertex. Tang and Chen [28] satisfied some interpolation requirements and minimized the developability change by a mesh deformation method and the method was further applied to cloth simulation [29]. In [7,30,6] efficient algorithms are given for finding the best (discrete) parameterization of an interpolating ruled surface for a range of optimization objectives, developability included. The algorithm given in [6] is based on the well-known Dijkstra's shortest path algorithm, is deterministic, and is able to find the global optimum. It will be a key component in our optimization algorithm. In the work of [8], a $G^0$ planar triangular strip can be constructed to support two curves with branches.

## 3. Problem description

### 3.1. Developability degree

For any two arbitrary space curves, there can be many ruled surfaces interpolating the two, of which the one that we are interested is the developable surface. Unfortunately, there may not exist such a desired interpolating developable surface unless the given curves meet some restricting geometric constraints (cf. [13]). For certain industries in which some materials such as metal and