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ScienceDirect

Computer-Aided Design 40 (2008) 447–454

COMPUTER-AIDED DESIGN

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An enhanced method and its application for fuzzy multi-criteria decision making based on vague sets[☆]

Kuo-Chen Hung[a,*], Gino K. Yang[a], Peter Chu[b], Warren Tsu-huei Jin[c]

[a] Department of Logistics Management, National Defense University, No. 70, Sec. 2, Jhongyang N. Road, Beitou District, Taipei City 112, Taiwan, ROC
[b] Department of Traffic Science, Central Police University, No. 56, Shujen Road, Takang Village, Kueishan Hsiang, Taoyuan County, Taiwan, ROC
[c] Defense Language Institute, No. 70, Sec. 2, Jhongyang N. Road, Beitou District, Taipei City 112, Taiwan, ROC

Received 31 July 2007; accepted 25 December 2007

Abstract

Ye [Ye Jun. Improved method of multicriteria fuzzy decision making based on vague sets. Computer-Aid Design 2007;39:164–9] presented an improved method to handle multi-criteria fuzzy decision-making problems based on vague set theory. He/She provided some functions to measure the degree of suitability of each alternative with respect to a set of criteria presented by vague values. However, in some cases, these functions do not give sufficient information about alternatives. Therefore, in this paper, an enhanced method is provided to measure the accuracy membership of each alternative so as to give additional information for the decision maker. In addition, to making computing and ranking results easier and to increase the recruiting productivity, a computer-based decision-support system is also developed, which may help to make a decision more efficiently.

Keywords: Fuzzy sets; Vague sets; Fuzzy multi-criteria decision making; Truth membership; False membership

1. Introduction

A general multi-criteria and evaluation process usually involves subjective assessments, resulting with imprecise data in qualitative manner. An engineering or management decision is generally made through available data and information that are mostly vague, imprecise, and uncertain, by nature. The decision-making process in engineering schemes, developed in the concept-designing phase, is one of these typical occasions, which usually need some methods to deal with uncertain data and information that are hard to define.

In designing phase, designers usually present many alternatives. However, the subjective characteristics of the alternatives are generally uncertain and need to be evaluated through decision maker's insufficient knowledge and judgments. The nature of this kind of vagueness and uncertainty is fuzzy rather than random, especially when subjective assessments are involved in the decision-making process. Fuzzy set theory offers a possibility for handling these sorts of data and information involving the subjective characteristics of human nature in the decision-making process.

The theory of fuzzy sets, proposed by Zadeh in 1965 [1], has been used for handling fuzzy decision-making problems [2–4]. Kickert [3] has discussed the field of fuzzy multi-criteria decision making. Zimmermann [5] illustrated a fuzzy set approach to multi-objective decision making, comparing it with other approaches to solve multi-attributive decision-making problems based on fuzzy set theory. Yager [6] presented a fuzzy multi-attributive decision-making method in terms of crisp weights. He [7] not only introduced ordered weighted aggregation operators but also found out their properties. Laarhoven and Pedrycz [8] presented a method for multi-attributive decision making using fuzzy numbers as weights.

Vague sets, proposed by Gau and Buehrer in 1993 [9], are a generalized form of fuzzy sets. These sets were used by Chen and Tan [10]. They present some new techniques for handling multi-criteria fuzzy decision-making problems based on vague set theory. The proposed techniques used a score function, S,

☆ This work was supported in part by the National Science Council of Taiwan, R.O.C. under Grant NSC 96-2416-H-606-001.

* Corresponding author. Tel.: +886 2 28986600x228358; fax: +886 2 28985927.
E-mail address: kuochen.hung@msa.hinet.net (K.-C. Hung).

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doi:10.1016/j.cad.2007.12.006

to evaluate the degree of suitability to which an alternative satisfies the decision maker's requirement. Besides, Hong and Choi [11] also presented an accuracy function, H, to replace the score function, S in Chen and Tan' method [10]. Based on the accuracy function, the degree of accuracy resulting from the grades of membership of the alternatives may satisfy the decision maker's requirements.

Ye [12] has recently proposed an improved method with evaluating function, J, by taking into account the effect of an unknown degree (hesitancy degree) of the vague values on the degree of suitability to which each alternative satisfies the decision maker's requirement. However, in some cases, these functions do not give sufficient information about alternatives.

The details of each of the above method will be depicted in Section 4. In this paper, an enhanced score function is proposed to evaluate the degree of suitability of each alternative by vague sets. Moreover, the proposed function can provide another useful way to assist the decision maker.

The remainder of this paper is organized as: In Section 2, the vague sets theory is briefly depicted. In Section 3, the general features of a multi-criteria decision-making problem are described. A review of the existing multi-criteria decision-making approaches by vague sets is made in Section 4. An enhanced score function to handle a multi-criteria fuzzy decision-making problem is presented in Section 5. The proposed method is applied to illustrate a case study and discussions in Section 6. A computer-based information interface is provided in Section 7 before conclusions are drawn.

2. Vague sets theory and its operations

In this section, the definitions and properties of vague sets are briefly introduced and the set operations of vague sets based on Gau and Buehrer [9] are defined.

Let X be the universe of discourse, $X = \{x_1, x_2, x_3, \dots, x_n\}$, with a generic element of X denoted by x_i. A vague set A in X is characterized by a truth-membership function t_A and a false-membership function f_A,

$$t_A : X \to [0, 1], \qquad f_A : X \to [0, 1],$$

where $t_A(x_i)$ is a lower bound on the grade of the membership of x_i derived from the evidence for x_i; $f_A(x_i)$ is a lower bound on the negation of x_i derived from the evidence against x_i and $t_A(x_i) + f_A(x_i) \le 1$. The grade of the membership of x_i in the vague set A is bounded to a subinterval $[t_A(x_i), 1 - f_A(x_i)]$ of $[0, 1]$. The vague value $[t_A(x_i), 1 - f_A(x_i)]$ indicates that the exact grade of the membership $u_A(x_i)$ of x_i may be unknown. However, it is bounded by $t_A(x_i) \le u_A(x_i) \le 1 - f_A(x_i)$, where $t_A(x_i) + f_A(x_i) \le 1$. Fig. 1 shows a vague set of real number R [13]. Moreover, when the universe of discourse U is discrete, $U = \{x_1, x_2, \dots, x_n\}$. A vague set V in U can be written as

$$V = \{(x_1, [t_V(x_1), 1 - f_V(x_1)]), (x_2, [t_V(x_2), 1 - f_V(x_2)]), \dots, (x_n, [t_V(x_n), 1 - f_V(x_n)])\},$$

where $x_i \in U$ for $i = 1, 2, \dots, n$.

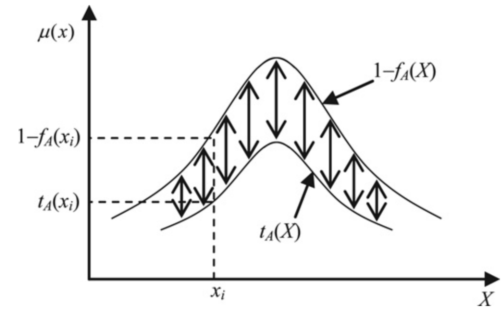

Fig. 1. Vague set explanation of real number R.

For example, let U be the universe of discourse, $U = \{1, 2, 3, 4, 5\}$. A vague set "Small" of U may be defined as

$$\text{Small} = \{(1, [1, 1]), (2, [0.9, 1]), (3, [0.6, 0.8]), (4, [0.3, 0.5]), (5, [0.1, 0.2])\}.$$

From the "Small" viewpoint, element "4" obtain a vague interval [0.3, 0.5], then we can see that $t_{\text{small}}(4) = 0.3$, $f_{\text{small}}(4) = 0.5$ and $u_{\text{small}}(4) = 0.2$. The result can be interpreted as "the vote for element "4" being small is 3 for pro, 5 for con and 2 for unknown". Similarly, element "1", "2", "3" and "5" can also be interpreted by the same way.

Definition 1 (*[10]*). The minimum operator of two vague sets, $A(x_i)$ and $B(x_i)$, is a vague set, $M(x_i)$, written as $M(x_i) = A(x_i) \wedge B(x_i)$. The truth-membership and false-membership functions are $t_M(x_i)$ and $f_M(x_i)$, respectively, where $\forall x_i \in X$, $t_M(x_i) = \min(t_A(x_i), t_B(x_i))$ and $1 - f_M(x_i) = \min(1 - f_A(x_i), 1 - f_B(x_i))$. That is,

$$[t_M(x_i), 1 - f_M(x_i)] = [t_A(x_i), 1 - f_A(x_i)] \wedge [t_B(x_i), 1 - f_B(x_i)] = [\min(t_A(x_i), t_B(x_i)), \min(1 - f_A(x_i), 1 - f_B(x_i))].$$

Definition 2 (*[10]*). The maximum operator of two vague sets, $A(x_i)$ and $B(x_i)$, is a vague set, $Z(x_i)$, written as $Z(x_i) = A(x_i) \vee B(x_i)$. The truth-membership function and false-membership functions are $t_Z(x_i)$ and $f_Z(x_i)$, respectively, where $\forall x_i \in X$, $t_Z(x_i) = \max(t_A(x_i), t_B(x_i))$ and $1 - f_Z(x_i) = \max(1 - f_A(x_i), 1 - f_B(x_i))$. That is,

$$[t_Z(x_i), 1 - f_Z(x_i)] = [t_A(x_i), 1 - f_A(x_i)] \vee [t_B(x_i), 1 - f_B(x_i)] = [\max(t_A(x_i), t_B(x_i)), \max(1 - f_A(x_i), 1 - f_B(x_i))].$$

3. Multi-criteria problem formulation by vague sets

The multi-criteria fuzzy decision-making problems usually involve a set of n units (alternatives) A_i $(i = 1, 2, \dots, n)$. These alternatives are to be evaluated based on a set of m criteria C_j $(j = 1, 2, \dots, m)$, which are independent of each other. Therefore, we can get the characteristics of the alternative A_i and criteria C_j presented as follows:

$$A_i = \{(C_1, [t_{iC_1}, (1 - f_{iC_1})]), (C_2, [t_{iC_2}, (1 - f_{iC_2})]), \dots, (C_m, [t_{iC_m}, (1 - f_{iC_m})])\}, \quad 1 \le i \le n \text{ and } 1 \le j \le m,$$