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Constrained 3D shape reconstruction using a combination of surface fitting and registration

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Abstract

We investigate 3D shape reconstruction from measurement data in the presence of constraints. The constraints may fix the surface type or set geometric relations between parts of an object's surface, such as orthogonality, parallelity and others. It is proposed to use a combination of surface fitting and registration within the geometric optimization framework of squared distance minimization (SDM). In this way, we obtain a quasi-Newton like optimization algorithm, which in each iteration simultaneously registers the data set with a rigid motion to the fitting surface and adapts the shape of the fitting surface. We present examples to show the applicability of our method to constrained 3D shape fitting for reverse engineering of CAD models and to high accuracy fitting with kinematic surfaces, which include surfaces of revolution (reconstructed from fragments of archeological pottery) and spiral surfaces, which are fitted to 3D measurement data of shells. Our optimization algorithm can combine registration of multiple scans of an object and model fitting into a single optimization process which is shown to be superior to the traditional procedure, which first registers the data and then fits a model to it.

Keywords: Constrained surface fitting; Registration; Squared distance minimization; Digital reconstruction

1. Introduction

The motivation for the present research comes from reconstruction of objects from 3D scanner data, where special kinematic surfaces (cones, cylinders, general surfaces of revolution, helical surfaces) appear frequently. Many reconstruction algorithms for the more general representatives of these surface classes require estimated surface normals [20,23]. Although these methods are quite efficient when good normal estimates are available, they lack the desired precision if it is difficult to obtain accurate normal estimation or the deviation of the data from the ideal shape model is relatively large; an example is the reconstruction of vessels from archeological findings. Moreover, in these methods the computation of the sweeping motion is separated from the computation of the swept profile, which is a further source of errors.

In the present paper, we extend recent work on improved reconstruction of surfaces of revolution [26] with a more generally applicable concept arising from the geometric optimization framework of *squared distance minimization* (SDM) [18,19,25,27,28]. Our new method combines the two types of optimization problems that have been solved so far with SDM, namely *curve/surface fitting* and *registration*. This new approach is not only applicable to surfaces of revolution but also to other classes of surfaces and to a number of surface reconstruction problems in reverse engineering in the presence of constraints.

1.1. Previous work

Since the focus of the present work is on *constrained* 3D shape reconstruction, we only review research in this direction. A constraint may fix the surface type: there have been a considerable number of contributions to fitting with special surfaces and thus we refer to [23] for a detailed survey. The existing methods are mainly taken from geometry (Gaussian image, line geometry, kinematical geometry), image processing (methods in extension of the Hough transform) and optimization (non-linear least squares problems). They are also used for surface type recognition (shape filters).

Fitting data with a surface of a given type that is determined with appropriate shape filters, while maintaining constraints

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between the individual elements of the surface, is a challenging problem [23]. Not only do we need to check the consistency of the constraints, we also need to fit the data simultaneously under these constraints. The work of Benkö et al. [2], Fisher [6], and Karniel et al. [10] can be considered to constitute the state of the art in this area. In the actual fitting part of the problem, most authors use a least squares formulation which embeds the constraints via penalty terms.

Our research is based on a combination of registration and fitting, and in this sense closely related to the work on knowledge based image segmentation via a combination of registration and active contours [15,17,24] and to deformable models introduced by Terzopoulos and Fleischer [21]. We also present a new solution for the simultaneous treatment of multiple view registration and model fitting, which extends prior work by Jin et al. [9] and Tubic et al. [22].

1.2. Contributions

Our contributions in this paper are:

- The extension of the SDM method to surface approximation with error measurement orthogonal to the fitting surface;
- The combination of registration and surface fitting within the SDM framework;
- Refined algorithms for fitting with kinematic surfaces (rotational, helical and spiral surfaces) plus a demonstration of their efficiency for shape reconstruction from measurement data of archeological pottery, shells and engineering objects;
- A new way of incorporating constraints into 3D surface reconstruction for applications in reverse engineering of CAD models;
- An efficient optimization algorithm which combines multiple view registration and model fitting and in this way achieves higher accuracy than the traditional approach which first registers the data and then fits a model to it.

2. Fundamentals of SDM

Here we summarize a few basic facts about squared distance minimization (SDM). For more details and issues of efficient implementation we refer to [1,18,19,25,27]. Before entering this discussion, we would like to point out that many authors have used the distance field [13,14] for registration and fitting; in fact, the concept of the distance field is so closely tied to the problem that it must occur in some way. However, most papers do not use the distance function in the same way as we are doing it: we use local quadratic approximants of the squared distance function and in this way obtain fast local convergence via algorithms of the Newton or quasi-Newton type.

2.1. Squared distance function of a surface

Given a surface $\Phi \subset \mathbb{R}^3$, the squared distance function d^2 assigns to each point $\mathbf{x} \in \mathbb{R}^3$ the square of its shortest distance to Φ. The importance of this function for our algorithms lies in the fact that we want to compute a surface, which minimizes the sum of squared distances to the data point cloud. Since several important optimization concepts require second order approximants of the objective function, we need to derive second order approximants of d^2.

Let us fix the notation. We consider a surface Φ with unit normal vector field $\mathbf{n}(\mathbf{s}) = \mathbf{n}_3(\mathbf{s})$, attached to points $\mathbf{s} \in \Phi$. At each point $\mathbf{s}$, we have a local Cartesian frame $(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n})$, where the first two vectors $\mathbf{n}_1$, $\mathbf{n}_2$ determine the principal curvature directions. We will refer to this local frame as the principal frame $\Pi(\mathbf{s})$. Let κ_j be the (signed) principal curvature in the principal curvature direction $\mathbf{n}_j$, $j=1,2$, and let $\rho_j = 1/\kappa_j$.

Let $\mathbf{s} \in \Phi$ be the normal foot point of a point $\mathbf{p} \in \mathbb{R}^3$, i.e., $\mathbf{s}$ is the closest point on Φ to $\mathbf{p}$. Expressed in the principal frame at $\mathbf{s}$ the second-order Taylor approximant F_d of the function d^2 at a point $\mathbf{x} \in \mathbb{R}^3$ in a neighborhood of $\mathbf{p}$ is:

$$F_d(\mathbf{x}) = \frac{d}{d-\rho_1}[\mathbf{n}_1 \cdot (\mathbf{x}-\mathbf{s})]^2 + \frac{d}{d-\rho_2}[\mathbf{n}_2 \cdot (\mathbf{x}-\mathbf{s})]^2 + [\mathbf{n}_3 \cdot (\mathbf{x}-\mathbf{s})]^2. \quad (1)$$

Here, $[\mathbf{n}_j \cdot (\mathbf{x}-\mathbf{s})]^2, j=1,2,3$, are the squared distances of $\mathbf{x}$ to the principal planes and tangent plane at $\mathbf{s}$, respectively.

In the important special case of $d=0$ (i.e., $\mathbf{p}=\mathbf{s}$), the approximant F_d equals the squared distance function to the tangent plane of Φ at $\mathbf{s}$. Thus, if $\mathbf{p}$ is close to Φ, the squared distance function to the tangent plane at $\mathbf{p}$'s closest point on Φ is a good approximant of d^2.

In a Newton-like iteration it is important to employ nonnegative quadratic approximants; we obtain them by removing from the expression of $F_d(\mathbf{x})$ in (1) those terms with a negative coefficient $d/(d-\rho_j)$; see [25].

2.2. Registration using SDM

A set of points $X^0 = (\mathbf{x}_1^0, \mathbf{x}_2^0, \ldots) \subset \mathbb{R}^3$ is given in some coordinate system Σ^0. It will be rigidly moved (i.e. registered) to be in best alignment with a given surface Φ, represented in another system Σ. We view Σ^0 and Σ as a moving system and a fixed system, respectively. A position of X^0 in Σ is denoted by $X = (\mathbf{x}_1, \mathbf{x}_2, \ldots)$. It is the image of X^0 under some rigid body motion α. Since we identify positions with motions and the motions have to act on the same initial position, we write $X = \alpha(X^0)$, or $\mathbf{x}_i = \alpha(\mathbf{x}_i^0)$.

The registration problem is formulated in a least squares sense [3,5]: Compute a rigid body transformation α^*, which minimizes the sum of squared distances of $\alpha(\mathbf{x}_i^0)$ to Φ:

$$F(\alpha) = \sum_i d^2(\alpha(\mathbf{x}_i^0), \Phi). \quad (2)$$

Starting from an appropriate initial position α^0, SDM performs a Newton-like iteration to minimize F [19]. We describe here a single iteration of the algorithm: Since F is the sum of squared distances of the data points $\mathbf{x}_i$ to the model