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New multiresolution modeling techniques in CAD

Jinxing Yin *, Guanlong Chen, Zhongqin Lin

School of Mechanical Engineering, Auto Body Manufacturing Technology Center (B.M.T.C), Shanghai Jiaotong University, Shanghai 200030, People's Republic of China

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Abstract

To apply wavelet transformation to CAD surface model composed of multiple surface patches, an algorithm called wavelet signal separation to preserve geometric constraints during wavelet transformation is proposed. The algorithm divides the B-spline control points into those associated and those unassociated with the geometric constraints. Through preserving the signal information associated with the constraint control points, the geometric constraints can be automatically preserved after wavelet transformation. This paper also briefly investigates two types of the detail feature propagation technique in wavelet-based multiresolution CAD system. One is detail feature motion. The other is detail feature repetition. Finally, a comprehensive example is presented to illustrate the effects of the combination of those techniques.

Keywords: Wavelet; Curve/surface modeling; Geometric constraints; Signal separation

1. Introduction

Wavelet-based multiresolution analysis (MRA) is based on decomposing a vector space into a set of nested vector spaces with different scales, and then analyzing the properties of functions in the time and frequency domains in those different scale spaces. The techniques are especially suitable for analyzing non-periodic signals and are widely used in different areas. Finkelstein and Salesin [1] in 1994 first introduced MRA based on B-spline wavelets to curve/surface modeling to facilitate a variety of multiresolution editing operations. MRA brought a novel conception to the area of curve/surface modeling. In current CAD systems there are many modeling methods, such as non-uniform rational B-splines (NURBS) modeling method, constructive solid geometry (CSG) modeling method, boundary representation (B-rep) modeling method, and recently free-form deformation (FFD) modeling method [2,3], partial differential equation (PDE) modeling method [4,5], energy functional optimization modeling method [6,7]. However, all the methods for object modeling are carried out in a single three-dimensional vector space. We define such style of CAD system as single-space CAD system.

* Corresponding author. Tel.: +86 21 62932125 0; fax: +86 21 62932125 103.

E-mail address: yjxiang@sjtu.edu.cn (J. Yin).

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Distinguishing from traditional single-space CAD system, MRA can edit curves/surfaces in different resolution spaces for different effects. We can call such a CAD system with MRA as multi-space CAD system. The multiresolution framework is attractive since the higher resolution details and the lower resolution (smoothed) curves are simultaneously available. Applications of MRA to curve/surface modeling have received wide attention in recent years and fruitful results have been obtained [8–20]. With multiresolution editing, the curve can be smoothed and the overall form of the curve can be changed while preserving its details (sweep editing). The curve can be edited at any continuous level of detail (fractional editing). Additionally, the curve's character can be changed without affecting its overall shape. However, to further complete multi-space CAD system, some other operations still need more attentions. For example, some well-known operations in tradition single-space CAD system as moving a well-defined detail feature from one location to another location or repeating a well-defined detail feature at some desired locations have not been clearly addressed in the literature for multi-space CAD system yet.

Another important issue is related to geometry constraints in multi-space CAD system. As well known, most of CAD surface models are consisted of multiple surface patches. How to keep geometrical continuity between different surface patches when MRA editing operation is applied still remains challenging. The end-point interpolating B-spline wavelet transformation developed in [9–11] is confined to a single tensor-product B-spline patch. MRA editing destroys the original geometric constraints associated with curves/surfaces. Takahashi [12,13] proposes an approach of designing shapes that imposes different constraints at multiple levels of resolution by using energy functions. Wu and Wang [14] propose a multilevel function constructed by combining multiresolution wavelet analysis with an energy function to satisfy geometric boundary constraints. Those approaches are to impose constraints through introducing the energy function by additional penalty terms associated with each of geometric constraints during wavelet transformation. It is known that with the introduction of additional energy functions, MRA editing becomes significantly complicated, especially in dealing with larger scale data [21]. Ranga and Ji developed an algorithm in Ref. [15] for multi-patch deformation by enforcement of G0 continuity between adjacent patches, and after editing G1 continuity may be recaptured by numerical optimization of the error vectors along boundary seams. Gershon Elber presents a scheme that combines multiresolution control with linear constraints into one framework, allowing one to perform multiresolution manipulation of nonuniform B-spline curves, while specifying and satisfying various linear constraints on the curves [16]. In this paper, we investigate a simpler and more efficient technique for keeping the constraints for multiple patches. An algorithm called wavelet signal separation to preserve geometric constraints during wavelet transformation in curve/surface modeling is proposed. The algorithm first divides the B-spline control points into those associated and those unassociated with the geometric constraints. Through preserving the signal information associated with the geometric constraints, the geometric constraints can be directly preserved after wavelet transformation.

This paper also briefly discuss detail feature propagation technique in multiple-space CAD system to extend the earlier work on multiresolution curve and surface operations [1,9–11, 15] such as sweep editing, fractional editing synthesis editing and detail blending. The considered propagation technique can be detail feature motion or detail feature repetition in same or different curve/surface patches. Detail feature motion is a way to move a well-defined detail feature from one location to another location. Detail feature repetition is a way to repetitively copy a well-defined detail feature on different desired locations.

The rest of the paper is organized as follows. MRA and endpoint interpolating B-spline wavelets used for representing curves and surfaces are introduced in Section 2. In Section 3, techniques for detail feature propagation are discussed. To apply wavelet transformation to multiple patches, the wavelet signal separation technique is presented in Section 4. Implementation examples are presented in Section 5 to demonstrate those techniques. Conclusions and future research directions are in Section 6.

2. MRA and endpoint interpolating B-splines

Wavelets were developed for the purpose of approximation by Daubechies in Ref. [17]. Mallat in Ref. [18] proposed a framework for wavelet-based MRA and expanded its application to signal processing. Recently, extensive attentions have been focused on MRA application to the field of computer graphics, see for example, Refs. [19,20]. This section provides a brief background on the theory of MRA and endpoint interpolating B-spline wavelets technique.

2.1. Multiresolution analysis

The basic concept in MRA is a set of nested vector spaces V^j such that:

$$V^0 \subset V^1 \subset V^2, \ldots, V^j \tag{1}$$

In the following, superscript j represents the scale of the vector space. As j increases, the vector spaces have a finer resolution.

The basis functions of space V^j are called scaling functions and are denoted by $\Phi_j(x)$. The orthogonal complement of V^j in V^{j+1} is denoted as W^j, i.e. $V^{j+1} = V^j + W^j$. The space V^j can be decomposed in a sequential manner as:

$$V^j = V^0 + W^0 + W^1 + \cdots + W^{j-1} \tag{2}$$

The basis functions in W^j are called wavelet functions and are denoted by $\Psi_j(x)$.

According to MRA, a function $f(x)$ at scale j denoted by $f(x) = \sum_{i=-\infty}^{+\infty} a_i \phi_i^j$ can be represented by a combination of lower scale functions and wavelets as

$$f(x) = \sum_{i=-\infty}^{+\infty} c(i)\phi_i^m(t) + \sum_{n-m}^{j} \sum_{k=-\infty}^{+\infty} \mathrm{d}(k,n)\varphi_k^n(t).$$

For orthogonal wavelets the coefficients $c(i)$ and $d(k,n)$ can be computed through the following equations: $c(i) = \int_{-\infty}^{+\infty} f(x)\phi_i^m(t)\mathrm{d}t$, and $\mathrm{d}(k,n) = \int_{-\infty}^{+\infty} f(x)\varphi_k^n(t)\mathrm{d}t$.

2.2. Endpoint interpolating B-spline wavelets

A function in the polynomial space V^j can be represented by a limited number of scaling functions as

$$f(x) = \Phi_j(x) \cdot C_j \tag{3}$$

where $\Phi_j(x)$ is a single row matrix formed by all the B-spline basis functions and the coefficients matrix, C^j by all the control points. Since V^j is a set of nested vector spaces, a matrix P^j exists such that

$$\Phi_{j-1}(x) = \Phi_j(x)P_j. \tag{4}$$

Similarly, since W^{j-1} is also a subspace of V^j, a matrix Q_j exists such that:

$$\Psi_{j-1}(x) = \Phi_j(x)Q_j \tag{5}$$

Eqs. (4) and (5) are often referred to as the scale relations of scaling functions $\Phi_j(x)$ and wavelets $\Psi_j(x)$. The matrices P_j and Q_j are called synthesis filters. Eqs. (4) and (5) can be combined