Computer Aided Geometric Design 43 (2016) 2–15

Contents lists available at ScienceDirect

Computer Aided Geometric Design

www.elsevier.com/locate/cagd

Surface fitting with cyclide splines

Pengbo Bo [a,*], Yang Liu [b], Changhe Tu [c], Caiming Zhang [c], Wenping Wang [d]

[a] Harbin Institute of Technology at Weihai, China
[b] Microsoft Research Asia, China
[c] Shandong University, China
[d] The University of Hong Kong, China

ARTICLE INFO

Article history:
Available online 23 February 2016

Keywords:
Dupin cyclide
Surface fitting
Cyclide spline surface
Principal patch

ABSTRACT

The cyclide spline surface is a G^1 smooth piecewise surface composed of Dupin cyclide patches, thus inheriting several favorable geometric properties of the Dupin cyclide, such as the closeness under offset operation. Due to the lack of shape flexibility of Dupin cyclides, it has been an outstanding problem to use a G^1 smooth piecewise Dupin cyclide surface to model a free-form shape. We solve this problem by proposing a novel method for approximating a given free-form smooth surface with a cyclide spline surface. The key to our solution is increasing the fitting flexibility of a cyclide spline by relaxing and optimizing both the vertices and surface frames of the cyclide spline simultaneously in a global manner. Furthermore, we apply patch subdivision for local surface refinement and use spherical patches to fill in umbilical regions. The effectiveness of our method is demonstrated on a variety of surface shapes.

1. Introduction

Surface fitting is a fundamental task in computer aided geometric design and computer graphics. Most existing works on surface fitting use NURBS surfaces or subdivision surfaces which are capable of approximating freeform surfaces with high accuracy (Piegl and Tiller, 1997; Litke et al., 2001). However, for many applications in CAD and architecture, fitting a free-form shape using a special class of surfaces is often demanded to reduce the fabrication cost. For instance, ruled surfaces and developable surfaces are most commonly used in the above applications. However, surface fitting with these special surfaces can often be very difficult because their special geometric properties pose many constraints and so resulting in inadequate degrees of freedom for achieving high quality surface fitting.

We are interested in surface fitting with a special class of surfaces — *Cyclide spline surfaces* which are G^1 smooth piecewise surfaces composed of surface patches on Dupin cyclides. The cyclide spline inherits many remarkable properties of Dupin cyclide that are suitable for computer aided design and manufacturing (Chandru et al., 1989). For example, (1) a quad cyclide patch is a *principal patch* (Martin, 1983) bounded by curvature lines which are all circular arcs; (2) the four vertices of a quad cyclide patch are co-circular; (3) any offset surface of a cyclide patch is still a cyclide patch.

These geometric properties of a cyclide patch facilitate many tasks in geometric modeling, such as curvature analysis, offset surface computation, and conversion of a cyclide surface into a rational polynomial form. However, previous attempts on surface fitting with cyclide splines suggest that the degree of freedom of cyclide spline surfaces is not large enough for

* Corresponding author.
E-mail addresses: pengbo@hitwh.edu.cn (P. Bo), yangliu@microsoft.com (Y. Liu), chtu@sdu.edu.cn (C. Tu), czhang@sdu.edu.cn (C. Zhang), wenping@cs.hku.hk (W. Wang).

http://dx.doi.org/10.1016/j.cagd.2016.02.018
0167-8396/

modeling freeform shapes (Srinivas et al., 1996). Essentially, all the previous methods try to solve for control variables of a fitting cyclide spline in a local, propagation manner and fail to avoid undesirable undulations of fitting surfaces.

Recently, some works demonstrate the modeling ability of cyclide patches in architecture modeling (Bo et al., 2011; Mesnil et al., 2015b; Krasauskas, 2011). In Bo et al. (2011), the first computational approach is proposed for simultaneously determining all control variables in a cyclide spline surface, and it is shown to be more powerful than previous propagation-based methods. However, the fitting ability of the cyclide spline surface is not fully explored in that work in that there is no exploration on local refinement and that umbilical regions on surfaces are not handled. In this paper, we report further studies on surface fitting with cyclide spline surfaces. Specifically, we introduce T-junctions in cyclide splines to allow adaptive surface subdivision for local surface refinement and integrate multi-sided spherical patches as special cyclide patches to fill umbilical regions.

1.1. Related work

Cyclide patches for surface modeling are first proposed and studied by Martin (1982, 1983). Since then, a number of methods have been proposed for modeling a free-form surface composed of quadrilateral cyclide patches in a column/row arrangement. The method in Srinivas et al. (1996) allows the user to edit some boundary vertices in a planar quadrilateral mesh which serves as the underlying structure of a surface composed of cyclide patches. The method in Martin et al. (1986) builds a circular mesh by starting from two crossing central polylines.[1] A method for creating tubular shapes with tubular cyclide patches is presented in Srinivas and Dutta (1994). Spherical images of cyclide patches are used in McLean (1985) for manipulating surface patches on the unit sphere. Krasauskas and Mäurer study cyclides in the 4-dimensional cyclographic model of Laguerre geometry (Krasauskas and Mäurer, 1999). Bobenko and Huhnen-Venedey study cyclide spline surfaces from the viewpoint of discrete differential geometry and extend them to volumetric splines (Bobenko and Huhnen-Venedey, 2012).

All these methods share a two-step pipeline in which (1) a circular quadrilateral mesh is first created; and (2) cyclide patches are constructed by first specifying the surface frame at a mesh vertex, and then determine the surface frames at the remaining vertices in a propagation manner. Note that a cyclide spline surface is uniquely determined by the surfaces frames at the vertices of the control mesh. Since the circular control mesh is fixed in these methods, they do not make full use of degrees of freedom of cyclide patches, and, consequently, there are only three degrees of freedom left for the cyclide spline, which is too limited for modeling complex shapes. Hence, only simple shapes, such as pipe surfaces, have been successfully generated using these methods.

Some recent works make considerable progress towards surface modeling using cyclides. Cyclide patches are recently considered as building panels in freeform architectural design. Mesnil et al. fix a circular mesh and create cyclide patches by minimizing a shape fairness functional with respect to surface frames (Mesnil et al., 2015b). Mesnil et al. study the application of cyclidic nets to generate high node congruence in freeform structures (Mesnil et al., 2015a). Krasauskas considers closing umbilical regions in a cyclide spline surface by recursively adding quadrilateral cyclide patches with T-junctions around the umbilical point (Krasauskas, 2011). However, by this method, there always exists a hole, however small, at an umbilical point.

Bo et al. develop the first approach to surface modeling with cyclide spline surface that relaxes mesh vertices as well as surface frames attached to the vertices in a unified optimization procedure (Bo et al., 2011). This approach makes the full use of the flexibility in cyclide spline surface, thus produces superior surface modeling results to the propagation approach. While the work of Bo et al. (2011) demonstrates the modeling capability of cyclide spline surface in freeform architectural design, the surface fitting quality of the cyclide spline surface is not the main concern because the reference surface there acts mostly as a design intent and aesthetic appearance of paneling cyclide patches is often achieved at the cost of approximation quality.

The cyclide spline surface is also closely related to the principal strip model that finds applications in freeform architecture and CNC machining (Pottmann et al., 2008). A surface composed of cyclide patches can be easily converted into principal strip models by discretization along one principal direction while keeping the smoothness in the other direction. Moreover, cyclide spline surfaces are suitable for fairness control because boundary curves of cyclide patches are curvature lines whose fairness is a direct evaluation of the fairness of surfaces. Note that the minimum variation surfaces energy is zero for Dupin cyclides (Joshi and Séquin, 2007).

1.2. Contributions

In the present paper, we propose several new techniques for surface fitting with the cyclide spline surface. Our main contributions include:

- Umbilical regions are approximated by a smooth cyclide spline surface composed of an assembly of a multi-sided spherical patch with several surrounding quadrilateral cyclide patches;
- T-junctions are introduced to allow subdivision of cyclide patches for local refinement.

[1] A circular mesh is a discrete mesh surface whose faces are quads with their four vertices being coplanar and co-circular.