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Kernel-based adaptive sampling for image reconstruction and meshing

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A B S T R A C T

This paper introduces a kernel-based sampling approach for image reconstruction and meshing. Given an input image and a user-specified number of points, the proposed method can automatically generate adaptive distribution of samples globally. We formulate the problem as an optimization to reconstruct the image by summing a small number of Gaussian kernels to approximate the given target image intensity or density. Each Gaussian kernel has the fixed size and the same energy. After the optimization, the samples are well distributed according to the image intensity or density variations as well as faithfully preserved the feature edges, which can be used to reconstruct high-quality images. Finally, we generate the adaptive triangular or tetrahedral meshes based on the well-spaced samples in 2D and 3D images. Our results are compared qualitatively and quantitatively with the state-of-the-art in image sampling and reconstruction on several examples by using the standard measurement criteria.

1. Introduction

Sampling and reconstruction are widely used in computer graphics and its applications, such as halftoning and stippling, point-based rendering, and geometry processing, etc. There are several excellent surveys that describe different algorithms and their used domains (Mitchell, 1990; Glassner, 1995; Dutre et al., 2006; Pharr and Humphreys, 2004; Zwicker et al., 2015). One of the most important properties of the sampling distributions is adapting to a given image, so that the number of points in a region is proportional to the image density as well as the positions of points do well preserve the object shapes in the image without aliasing artifacts. Another desirable property is possessing spectral characteristics, i.e., blue noise property, such as Poisson disc samplings (Deussen et al., 2000), Lloyd's method for weighted Voronoi stippling (Secord, 2002), a variant of Lloyd's method by using capacity-constrained Voronoi tessellation (CCVT) (Balzer et al., 2009), an interacting particle model for electrostatic halftoning (Schmaltz et al., 2010), variational blue noise sampling (Chen et al., 2012), etc. Recently, Fattal (2011) presented a kernel density approach for generating point sets with high-quality blue noise properties that formulates the problem using a statistical mechanics interacting particle model. However, in the Fattal's method, users cannot explicitly specify the final sampling population since each kernel has a different size and the final sampling density is controlled by a statistical mechanism with local interactions. Thus if the initialization does not have a good estimation of target density, it will obtain a poor result. Besides that, all the above previous sampling methods

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mainly focus on blue noise characteristics without considering the fidelity of the sampling result to the original image, and the reconstructed image quality.

In this paper, we propose a method to minimize a total energy that is the difference between the total summation of a user-specified number of Gaussian energies and the given input image, leading to an adaptive distribution of samples, i.e., the Gaussian kernel centers, conforming to the input image intensity or density, which is determined by different applications. For instance, image intensity is used in image reconstruction; image density is used in stippling; and both of them can be used in adaptive mesh generation. Each Gaussian kernel has the fixed size and the same energy, enforcing the equal importance of each sample in the distribution. Then, the formula for the explicit gradient of the proposed energy function is derived and an efficient numeric approach is proposed to optimize the energy function by using a fast local search based on the L-BFGS method (Liu and Nocedal, 1989). After the optimization, the samples are well distributed according to the image intensity or density variations, i.e., densely in the high intensity or density regions and sparsely in the low intensity or density regions. Since each sample endowed with a fixed Gaussian energy, we can obtain the reconstructed image by summing all these sampling kernels at the optimized positions together. Finally, the adaptive triangular or tetrahedral meshes are generated based on the well-spaced samples in 2D and 3D images.

This paper makes the following contributions for generating high-fidelity adaptive samplings resulting in computing high-quality images and meshes:

- It introduces a new kernel-based approach and consists in optimizing all the samples with kernel energies globally without explicit control of sample population, so that it is sufficiently simple and efficient. The key idea is derived from reconstructing a high-quality image by using the sparse sampling points with kernel energies. When the total energy is optimized, the samples in the image domain will achieve the adaptive pattern with the desired input image intensity or density, as well as faithfully preserve the image feature edges. Since the samples have regular/adaptive patterns in the constant or linear intensity variation regions, and meanwhile well capture the sharp image features, i.e., non-linear intensity variation regions, we can generate high-quality adaptive triangular or tetrahedral meshes based on the well-spaced samples in 2D and 3D images.
- It presents a computationally feasible and efficient method for our energy optimization (Sec. 3). The computational complexity is $O(n+m)$, where n is the number of sampling points, and m is the number of pixels in the image domain. Such energy is C^{∞} smoothness and energy optimization strategy demonstrates very fast convergence speed, without any need for the explicit control of sampling population (e.g., dynamically inserting or deleting samples to meet the input given image intensity or density).

2. Related work

There are extensive studies in the literatures about the sampling distribution and its applications in image reconstruction. We mainly review the most related previous works to our study.

2.1. Point distribution and sampling

In computer graphics, sampling distribution on the image space has become an interesting research topic in the past few decades. Poisson disk samplings (Dippé and Wold, 1985) are uniformly distributed in space without overlapping between disks according to the predefined radius and have good spectrally sampling patterns. Cook (1986) proposed a dart throwing algorithm to generate point distributions. Deussen et al. (2000) generated the stipple drawings by using Poisson disc distributions. Liang et al. (2015) proposed a Poisson disk sampling algorithm based on disk packing for image stippling. Xing et al. (2014) presented highly parallel algorithms for remeshing polygonal models based on the sampling points from human visual perception. In the recent decades, Lloyd's (1982) method is a traditional method to compute centroidal Voronoi tessellation (CVT) for the uniform sampling distribution. McCool and Fiume (1992) improved the spectral properties of the results from Lloyd's method, but without explicit termination criteria. Moreover, Lloyd's method can provide a facility to generate the density sampling by specifying a target density function by Secord (2002). In Balzer et al.'s (2009) work, they proposed a variant of Lloyd's method by imposing a capacity constraint on the Voronoi tessellation. Schmaltz et al. (2010) generated halftoning using an interactive particle system inspired by the physical principles of electrostatics. The computational complexity of the above methods is very high as compared in Sec. 7.1. Chen et al. (2012) proposed an efficient variational framework based on an energy function combining the CVT energy and the CCVT energy. Zhong et al. (2013) used the Gaussian kernel to define an inter-particle energy and forces for surface meshing. Their method is formulated based on minimizing the total particle energy and our proposed method is to minimize the total reconstruction errors; besides that, two methods have different applications. Recently, Fattal (2011) presented a kernel density model for generating point sets with blue noise properties that formulates the problem using a statistical mechanics interacting particle model. However, in the Fattal's method, users cannot explicitly specify the final sampling population and the final density is controlled by a statistical mechanism with local interactions. Thus if the initialization of the sampling does not have a good estimation of target density, it will obtain a poor result. Furthermore, his method cannot well capture the image sharp features and we have compared with our method in Sec. 7.2. Generally, our kernel-based approach is different and consists in optimizing all the samples globally without explicit control of sample population, so that it makes the computation simple and efficient.