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Novel and efficient computation of Hilbert–Huang transform on surfaces

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ABSTRACT

Hilbert–Huang Transform (HHT) has proven to be extremely powerful for signal processing and analysis in 1D time series, and its generalization to regular tensor-product domains (e.g., 2D and 3D Euclidean space) has also demonstrated its widespread utilities in image processing and analysis. Compared with popular Fourier transform and wavelet transform, the most prominent advantage of Hilbert–Huang Transform (HHT) is that, it is a fully data-driven, adaptive method, especially valuable for handling non-stationary and nonlinear signals. Two key technical elements of Hilbert–Huang transform are: (1) Empirical Mode Decomposition (EMD) and (2) Hilbert spectra computation. HHT's uniqueness results from its capability to reveal both global information (i.e., Intrinsic Mode Functions (IMFs) enabled by EMD) and local information (i.e., the computation of local frequency, amplitude (energy) and phase information enabled by Hilbert spectra computation) from input signals. Despite HHT's rapid advancement in the past decade, its theory and applications on surfaces remain severely under-explored due to the current technical challenge in conducting Hilbert spectra computation on surfaces. To ameliorate, this paper takes a new initiative to compute the Riesz transform on 3D surfaces, a natural generalization of Hilbert transform in higher-dimensional cases, with a goal to make the theoretic breakthrough. The core of our theoretic and computational framework is to fully exploit the relationship between Riesz transform and fractional Laplacian operator, which can enable the computation of Riesz transform on surfaces via eigenvalue decomposition of Laplacian matrix. Moreover, we integrate the techniques of EMD and our newly-proposed Riesz transform on 3D surfaces by monogenic signals to compute Hilbert spectra, which include the space-frequency-energy distribution of signals defined over 3D surfaces and characterize key local feature information (e.g., instantaneous frequency, local amplitude, and local phase). Experiments and applications in spectral geometry processing and prominent feature detection illustrate the effectiveness of the current computational framework of HHT on 3D surfaces, which could serve as a solid foundation for upcoming, more serious applications in graphics and geometry computing fields.

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1. Introduction and motivation

With the rapid development of data acquisition devices and computing resources, diverse types of data are acquired routinely at an accelerated pace everyday. Novel data modeling, processing, and analysis methods must be developed in concert to address new technical challenges, and this is especially urgent when we are facing the *big data* challenge at present.

Hilbert–Huang transform (HHT), a fully data-adaptive multiscale data analysis and processing method, had been proposed initially by Huang et al. in 1998. Ever since its inception, it has gained tremendous popularity, and hence has proven to be extremely powerful for signal processing and analysis in 1D time series. As a result, it has yielded a wide variety of applications in many science and engineering fields (Huang and Wu, 2008), such as geophysics (Battista et al., 2007), atmospheric (Lundquist, 2003), climate (Di et al., 2014), oceanographic studies (Gemmrich and Farmer, 2004), geology (Chiang et al., 2010), and many more. The generalization of HHT to higher-dimensional, regular, tensor-product domains (e.g., 2D and 3D Euclidean space) has also demonstrated its widespread utilities in handling higher-dimensional data towards more quantitative processing and more accurate analysis. So far, typical applications include (but not are limited to): image compression (Ge and Yu, 2013; Linderhed, 2002; Tian et al., 2011), image analysis (Nunes et al., 2003; Xu et al., 2009), image fusion (Yeh, 2012), facial emotion recognition (Ali et al., 2015), skeleton pruning (Krinidis and Krinidis, 2013), field simulation and modulation (Gao et al., 2013; Ren et al., 2013), etc.

Compared with traditional techniques such as Fourier analysis, short-time Fourier analysis, and wavelet analysis, the most prominent advantage of HHT is that, it is a fully data-driven, adaptive method, especially valuable for processing and analyzing non-stationary and nonlinear signals (Huang et al., 1998). Its initial success in handling 1D time series clearly exhibits its potential in data analysis, and its strength is also echoed by its rapid generalization to image domains. The core technical elements of HHT are Empirical Mode Decomposition (EMD) and Hilbert spectral analysis. The goal of EMD is to decompose a signal into a finite number of Intrinsic Mode Functions (IMFs) with multi-scale oscillatory modes from fine to coarse and a residue with monotonic trend (Huang and Wu, 2008). Such IMFs as bases are global, completely data-adaptive and they differ substantially among different signals. It may be noted that, through the use of the sifting process, EMD only offers a set of empirical bases that are totally data-driven and tend to be much fewer than other well-known bases (e.g., Fourier bases, wavelet bases) (Huang et al., 1998; Huang and Wu, 2008). In order to glean more meaningful local information towards more quantitative analysis from such IMFs after EMD is carried out, we must resort to Hilbert transform (Gabor, 1946; Hahn, 1996), that essentially computes the time/space-frequency-energy distribution of a signal from each IMF (i.e., Hilbert spectra). Without Hilbert transform, we will be unable to pinpoint local frequency, energy, and phase information precisely. In technical essence, HHT's unique strength towards more quantitative and more accurate analysis results from its superior capability to reveal both global information (i.e., Intrinsic Mode Functions (IMFs) enabled by EMD) and local information (i.e., the computation of instantaneous frequency, phase, and energy information enabled by Hilbert spectral analysis) from any input signal (Huang et al., 1998; Huang and Wu, 2008).

Despite HHT's rapid development in processing time series and images in 2D/3D domain in the past decade, its theoretic advancement and applications on surface signals are hampered by our inability of conducting Hilbert spectra computation on surfaces. In general, HHT on surface signals remain severely under-explored so far (Mandic et al., 2013). The primary reason lies in the fact that 3D surfaces are irregular, curved, and possibly have arbitrarily complex structure which will bring forth much more challenges for the computation of the Hilbert transform on surfaces (Hahn, 1996). As a result, prior graphics-related efforts of HHT on surfaces mainly concentrate on identifying multiple intrinsic scales of the input signal via EMD in a global sense (Hu et al., 2014; Qin et al., 2009; Wang et al., 2012) or perform HHT computation via dimensionality reduction (Wang et al., 2015), and without an accurate and effective way to extract local information from each IMF, we are primarily confined to geometry signal processing such as denoising, smoothing, and enhancement.

Key contributions of this paper are as follows. First, its overarching contribution is primarily at the theoretic front. Specifically, we tackle the challenge of computing the Riesz transform on 3D surfaces, a natural generalization of Hilbert transform in higher-dimensional cases, with an ambitious goal of realizing the full potential of Hilbert–Huang Transform (HHT) in graphics and visual computing fields. Our computational solution is to take the full advantage of the relationship between Riesz transform and fractional Laplacian operator, which can facilitate the computation of Riesz transform on surfaces via eigenvalue decomposition of Laplacian matrix. Furthermore, we integrate our newly-developed theory and computational techniques in this paper with the feature-sensitive EMD for surface signals (Hu et al., 2014; Wang et al., 2012) to compute a Hilbert spectrum that contains the space-frequency-energy distribution of the signals at every location across the entire 3D surface domain (Fig. 1). This paper showcases our initial research endeavor in making HHT on surfaces computationally feasible. Second, at the application front, we explore a small subset of graphics applications to illustrate the effectiveness of the novel HHT theory on 3D surfaces. Based on our extensive experiments on popular geometric models, we are hoping to demonstrate that the Hilbert–Huang transform is invaluable for 3D surface modeling, processing, and analysis. In this paper, our theoretic undertakings would move us one step closer to our ultimate goal, which is to continue to broaden the horizon of HHT's powerful applications in relevant visual computing areas.