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Efficient Voronoi diagram construction for planar freeform spiral curves

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ABSTRACT

We present a real-time algorithm for computing the Voronoi diagram of planar freeform piecewise-spiral curves. The efficiency and robustness of our algorithm is based on a simple topological structure of Voronoi cells for spirals, which also enables us a direct construction of Voronoi structure without relying on intermediate polygonal or biarc approximations to the given planar curves. Using a Möbius transformation, we provide an efficient search for maximal disks. The correct topology of Voronoi diagram is computed by sampling maximal disks systematically, which entails subdividing spirals until each belongs to a pair/triple of spirals under a certain matching condition. The matching pairs and triples serve as the basic building blocks for bisectors and bifurcations, and their connectivity implies the Voronoi structure. We demonstrate a real-time performance of our algorithm using experimental results including the medial axis computation for planar regions under deformation with non-trivial self-intersections and the Voronoi diagram construction for disconnected planar freeform curves.

1. Introduction

Voronoi diagram is a partition of space into cells, each containing points closer to a specific site than to any other sites (Aurenhammer, 1991; Okabe et al., 2000). Common boundaries of adjacent Voronoi cells can be used for the construction of Medial Axis Transform (MAT) or Skeleton, a powerful shape descriptor originally introduced by Blum (1967, 1973). Nevertheless, the skeleton may not always be constructed by removing some redundancies from the common boundaries. (There may be certain skeletal parts missing from the Voronoi edges when some non-convex sites have self-bisectors in the interior of their Voronoi cells.) To avoid this problem, we consider sites which are spirals. (Spiral curve is a curvature monotone planar curve with no inflection point in the curve interior Barton and Elber, 2011; Meek and Walton 1995, 1999.) Spirals provide many useful geometric properties for the development of efficient and robust algorithms for planar curves (Aichholzer et al. 2009, 2010; Lee et al. 2015a, 2015b; Oh et al., 2012).

Regarding the construction of Voronoi diagram and medial axis for planar freeform curves, the segmentation into spirals makes the computation of topological structures very stable (Aichholzer et al. 2007, 2009, 2010; Aigner, 2007). Motivated by the theoretical guarantee on the convergence of the medial axis for spiral biarc spline approximation to the exact medial

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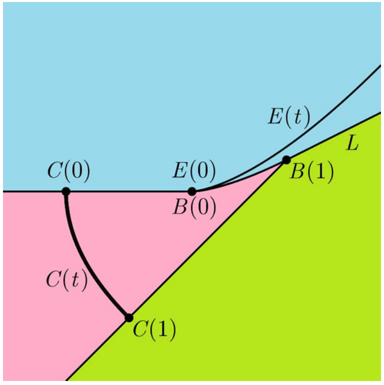

Fig. 1. A spiral curve $C(t) = \left(\frac{1}{2}t^2, -t\right)$, $0 \le t \le 1$, and its Voronoi diagram: the regions of different color correspond to the Voronoi cells of $C(0)$, $C(1)$, and the curve interior. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

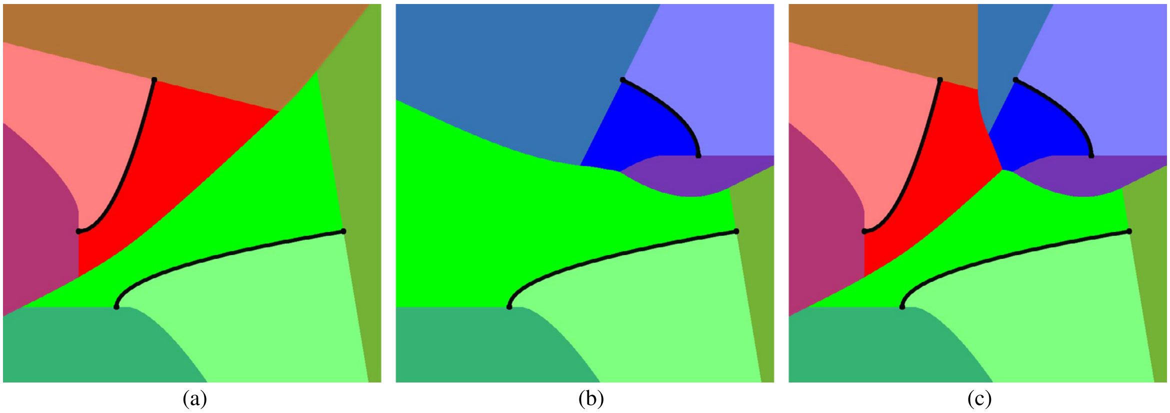

Fig. 2. Voronoi diagram for three spiral curves and their endpoints. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

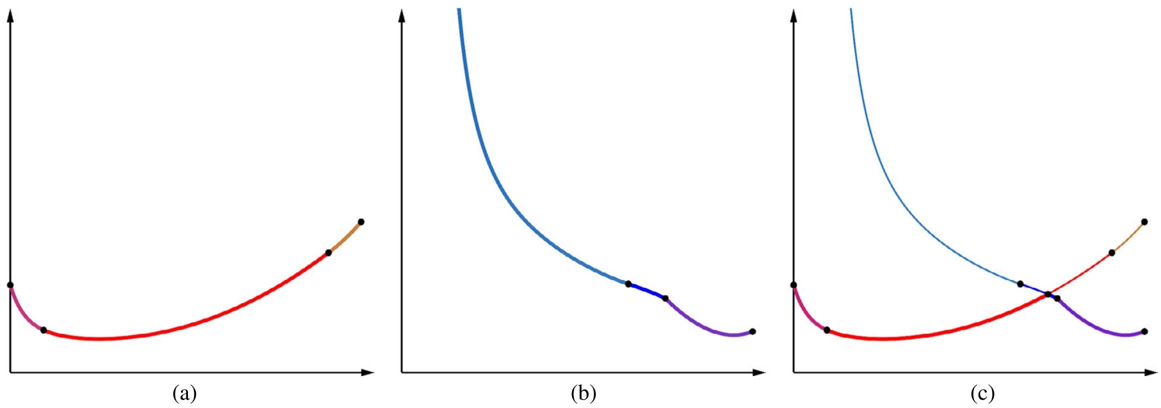

Fig. 3. Lower envelope of distance maps.

axis of the original curve (Aichholzer et al., 2007; Aigner, 2007), we develop an algorithm that accelerates the construction of Voronoi cells for planar spiral curves by directly computing the correct topology of Voronoi diagram using a small number of properly sampled maximal touching disks to the original freeform curves. The theoretical guarantee on the correct topology (in a finite number of sampling steps) also leads to the efficiency and robustness of our algorithm in practice. The bifurcation locations can be computed precisely using a numerical method (Hu and Wallner, 2005). The sequence of bisectors connecting bifurcations and terminals can be constructed very efficiently (within a given Hausdorff error bound) using the conic bisectors between spiral biarc approximations to the given planar curves (Aichholzer et al. 2007, 2009, 2010; Aigner, 2007).