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On the dimension of spline spaces over T-meshes with smoothing cofactor-conformality method ☆

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ABSTRACT

This paper provides a general formula for the dimension of spline space over general planar T-meshes (having concave corners or holes) by using the smoothing cofactor-conformality method. We introduce a new notion, the diagonalizable T-mesh, where the dimension formula is only associated with the topological information of the T-mesh. A necessary and sufficient condition for characterization of the diagonalizable T-mesh is also provided. By this new notion, we obtain some new dimension results for the spline spaces over T-meshes.

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1. Introduction

NURBS (Non-Uniform Rational B-Spline) is the de facto standard that is used to generate and represent free-form curves and surfaces in CAD (Farin, 2002). It is also a desirable tool for isogeometric analysis (Cottrell et al., 2009). A well-known and significant disadvantage of NURBS is that it is based on a tensor-product structure that uses a global knot insertion operation. It is necessary to generalize NURBS space to spline space which can handle T-junctions, or hanging nodes.

Many researchers have attempted this issue and several different methods have been developed over the years, including Hierarchical B-splines (Forsey and Bartels, 1988; Kraft, 1997; Vuong et al., 2011; Giannelli et al., 2012), T-splines (Sederberg et al., 2003, 2004; Li et al., 2012), Polynomial Splines over T-meshes (Deng et al., 2006, 2008), and LR B-splines (Johannessen et al., 2014; Mourrain, 2014). In order to apply the above locally refinable splines to isogeometric analysis (Bazilevs et al., 2010), a very important topic is to unravel the corresponding spline spaces which have important implications in establishing approximation, stability, and error estimates (Bazilevs et al., 2006). For example, Giannelli and Jüttler (2013) study the completeness of hierarchical B-spline space by calculating the dimension of spline space over some hierarchical T-meshes. Analysis-suitable T-spline spaces (Li et al., 2012; Scott et al., 2012; Li and Scott, 2014) are discovered by using the dimension result of spline space over another T-mesh. The LR B-spline space is characterized according to the dimension formula in Mourrain (2014). Thus, in order to understand the locally refinable spline spaces, one common, foundational, but non-trivial step is to calculate the dimension of the spline space over some T-meshes.

There are several different approaches to calculate the dimension of the spline space over some T-meshes, including the B-net method (Deng et al., 2006), the minimal determining set method (Schumaker and Wang, 2011), the smoothing

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cofactor-conformality method (Li et al., 2006) and the homological technique (Mourrain, 2014). The present paper focuses on the smoothing cofactor-conformality method. Because the smoothing cofactor-conformality method can convert the smoothness conditions into algebraic forms, we directly focus on the algebraic forms and study the matrix. If the matrix can be formed into a block upper triangular matrix, the dimension of the linear system is not associated with the knot values. Then, we use the condition to find a new notion, (diagonalizable T-mesh), regarding the corresponding T-meshes. A similar notion, (weighted T-mesh, Definition 3.8 in Mourrain, 2014), is independently discovered in Mourrain (2014), where the dimension (Theorems 3.3 and 3.9) is able to be computed in an explicit formula by using the homological techniques. Although the weighted T-mesh is associated with a predefined order for the l-edges, it can also be generalized to the T-mesh when the order for the l-edges exists, which is the same as the diagonalizable T-mesh in the present paper. The other difference of the present paper is that we provide a necessary and sufficient condition for characterization a diagonalizable T-mesh, which can be used to find some new results for the spline space with reduced smoothness (Deng et al., 2006; Schumaker and Wang, 2012a), and when the T-mesh has a sufficient amount of mono-vertices (the definition is in section 2). The present paper is also a generalization of the first dimension of spline space over T-mesh paper with smoothing cofactor-conformality method (Li et al., 2006). There are two main difference between these two papers. Firstly, we generalize the result to handle more general type of T-meshes. Secondly, the notion of diagonalizable T-mesh doesn't require the pre-defined order of l-edges while the papers (Li et al., 2006; Li and Chen, 2011a) are based on the predefined order of l-edges. In summary, the main contributions of the present paper include,

- We provide a general formula for the dimension of the spline space over the T-mesh (Theorems 3.1 and 3.3);
- We provide a new notion, the diagonalizable T-mesh, where the dimension formula is only associated with the topological information of the T-mesh. We also provide a necessary and sufficient condition that is required characterize diagonalizable T-meshes (Theorems 4.4 and 4.5);
- We provide new dimension results for the spline space over T-meshes that do not have a nested structure (in Section 5).

1.1. Prior work

Locally refinable splines. Four typical approaches have developed to generalize NURBS space to spline space which can handle T-junctions: the hierarchical B-splines, the T-splines, the splines over T-meshes and the LR B-splines. The hierarchical B-splines are originally introduced in 1988 by Forsey and Bartels (1988), which can be locally refined by using overlays. Then, Kraft suggests a selection mechanism for hierarchical B-splines (Kraft, 1997) to ensure their linear independence. Kraft's construction is further elaborated in Vuong et al. (2011), Giannelli et al. (2012) for the application in isogeometric analysis. T-splines are collections of B-spline functions that are defined on a T-mesh (T-grid) (Sederberg et al., 2003, 2004). T-splines are compatible forwards and backwards with NURBS and they have several advantages over NURBS, including local refinement (Sederberg et al., 2004; Scott et al., 2012) and watertightness (Sederberg et al., 2008). These capabilities make T-splines attractive for both CAD and isogeometric analysis (Bazilevs et al., 2010). Spline space over a T-mesh, or $\mathcal{S}(d_1, d_2, \alpha, \beta, \mathcal{T})$, is first introduced in Deng et al. (2006), which is a bi-degree (d_1, d_2) piecewise polynomial spline space over T-mesh $\mathcal{T}$ with smoothness orders α and β in two directions. Later, this spline is applied in adaptive fitting (Deng et al., 2008), stitching (Li et al., 2007), simplification (Li et al., 2010), and isogeometric analysis (Nguyen-Thanh et al., 2011; Wang et al., 2011), as well as for solving elliptic equations (Tian et al., 2011). The concept is also generalized to spline space over triangulations with hanging nodes (Schumaker and Wang, 2012b). Locally refinable splines (LR-splines) are proposed by Johannessen et al. (2014), with the refining process based on hand-in-hand LR-refinement which starts from a tensor-product mesh.

Dimension of spline spaces over T-meshes. In 2006, Deng et al. (2006) study the dimension of the spline space under certain constraint that the order of smoothness is less than half of the degree of the spline functions. If the T-mesh has no cycles, Deng et al. (2006) provide the explicit dimension formula. Schumaker and Wang (2012a) and Li et al. (2006) also give the result by using the minimal determining set method and the smoothing cofactor-conformality method respectively. Later, Buffa et al. (2012) analyze a special T-spline with reduced smoothness by using the dimension formula in Deng et al. (2006). In 2011, Li and Chen (2011b) discover that the dimension of the associated spline space is instable over particular T-meshes, (i.e, the dimension is not only associated with the topological information of the T-mesh but it is also associated with the geometric information of the T-mesh). Additionally, Berdinsky et al. (2012) provide two more examples of spline spaces $\mathcal{S}(5, 5, 3, 3, \mathcal{T})$ and $\mathcal{S}(4, 4, 2, 2, \mathcal{T})$ regarding the instability of the dimension. Mourrain (2014) provides a general formula for the spline spaces by using the homological techniques. Unfortunately there is a term in the dimension formula that is very difficult to compute in practice. Thus, they define a class of T-meshes, the weighted T-meshes, over which the dimension can be computed in an explicit formula. The weighted T-mesh is associated with the order of the l-edges. Wu et al. (2013) provide the dimension for spline space $\mathcal{S}(d, d, d-1, d-1, \mathcal{T})$ over a special hierarchical T-mesh by using the homological algebra technique. Deng et al. (2013), Zeng et al. (2015) derive a dimension formula for C^1 biquadratic and C^2 bi-cubic spline spaces over hierarchical T-meshes. Giannelli and Jüttler (2013) provide certain conditions that were necessary for the hierarchical spline basis to span the entire space of the piecewise polynomial functions defined on the underlying grid with a given degree and smoothness.

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