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Reconstruction of free-form space curves using NURBS-snakes and a quadratic programming approach ☆

Deepika Saini *, Sanjeev Kumar, Tilak Raj Gulati

Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee 247667, India

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ABSTRACT

In this study, we propose a robust algorithm for reconstructing free-form space curves in space using a Non-Uniform Rational B-Spline (NURBS)-snake model. Two perspective images of the required free-form curve are used as the input and a nonlinear optimization process is used to fit a NURBS-snake on the projected data in these images. Control points and weights are treated as decision variables in the optimization process. The Levenberg–Marquardt optimization algorithm is used to optimize the parameters of the NURBS-snake, where the initial solution is obtained using a two-step procedure. This makes the convergence faster and it stabilizes the optimization procedure. The curve reconstruction problem is reduced to a problem that comprises stereo reconstruction of the control points and computation of the corresponding weights. Several experiments were conducted to evaluate the performance of the proposed algorithm and comparisons were made with other existing approaches.

1. Introduction

The three-dimensional (3D) reconstruction of a space curve from its arbitrary perspective images is a core problem in computer vision. Many algorithms have been proposed for the 3D reconstruction of point features, where the scene is represented by a point cloud (Piegel, 1991; Flöry, 2009; Tomasi, 1992). However, model-based approaches that rely on the reconstruction of basic shapes such as lines, circles, and ellipses are more effective for understanding a 3D scene (Hartley, 1994; Piegel, 1991). The main reasons for using these primitive shapes are their availability and easy detection in images. The property of shape invariance under perspective projection makes the process of 3D reconstruction unambiguous for these primitive shapes. However, free-form curves and many complex objects cannot be represented easily using an analytical form, which makes the reconstruction of free-form curves and complex objects more difficult.

The Non-Uniform Rational B-Spline (NURBS) is a well-known mathematical technique, which addresses the problems of free-form shape representation (Farin, 1992; Piegel, 1991). NURBS has several properties that make it a suitable candidate for representing any shape or structure in two-dimensional (2D) and 3D geometry. With NURBS, the same representation can be used for 2D and 3D cases. Moreover, it can be determined completely by its control points in 2D and 3D cases. This implies that the problem of curve reconstruction in a 3D space may be treated as the reconstruction of a sparse set of points (i.e., control points). In Terzopoulos and Qin (1994), a new generalization of NURBS was proposed called dynamic

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* Corresponding author.
E-mail addresses: sainddma@iitr.ac.in (D. Saini), malikfma@iitr.ac.in (S. Kumar), trgorfma@iitr.ac.in (T.R. Gulati).

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Table 1
Symbols and their meaning in the present study.

Roman	
C	Non-uniform rational B-spline curve in 3D space
u	Independent variable of basis functions
B	Normalized B-spline basis function
P	Control points in 3D space
W	Weight corresponding to the 3D control point
k	Degree of B-spline basis function
$N+1$	Total number of control points
R	Rational B-spline basis function in 3D space
c	Non-uniform rational B-spline curve in 2D space
p	Control points in 2D space
w	Weight corresponding to the 2D control point
r	Rational B-spline basis function in 2D space
E_{int}	Internal energy of planar NURBS curve $c(u)$
$\frac{\partial}{\partial u}, \frac{\partial^2}{\partial u^2}, \frac{\partial^3}{\partial u^3}$	First, second, and third order partial derivatives with respect to parameter u
E_{ext}	External energy of planar NURBS curve $c(u)$
p^x, p^y	(x, y) coordinate of 2D control point
(x', y')	Unorganized data point
n	Total number of unorganized data points
t	Degree of derivative
Q	Residual function
d	Unorganized data points
f	Residual vector
J	Jacobian matrix
m	Number of iterations
Δp	Increment of parameter
$-J(p)^T Q$	Negative of gradient vector
c_s^x, c_s^y	Value of 2D NURBS curve along x and y axes at each parameter value
p_x, p_y	Homogeneous vector of 2D control points along x and y axes
D_x, D_y	Diagonal matrices containing the values of 2D NURBS curve along x and y axes
M	$(N+1) \times (N+1)$ symmetric and nonnegative matrix
b	Binormal vector
T^L, T^R	Left and right projection matrices
F^{RL}	Fundamental matrix
Greek symbols	
ξ	Knot
ζ	Knot vector
$\alpha, \beta, \gamma, \eta$	Constant parameters
λ	Damping factor
κ	Curvature function
$\kappa^{\max}$	Maximum curvature of 2D NURBS curve $c(u)$
ς	Constant parameter

NURBS (D-NURBS). The D-NURBS model is governed by dynamic differential equations, where it evolves the control points and weights continuously in response to the applied force when integrated numerically over a time interval.

The problem of 3D reconstruction involves finding a curve in space, where its projection fits the given data in two images with the minimum error. To achieve this, we require a correspondence between the data in the left and right images, which is called stereo matching. An energy minimization-based spline 'snake' is quite useful for establishing this correspondence (Kass et al., 1988; Menet et al., 1990; Wang and Cohen, 1994). Basically, the shape of the snake is controlled by internal and external forces. An energy function defined by a linear combination of these forces is then minimized iteratively by taking a user-specified initial estimate (initial configuration). The internal energy describes the continuity of the required curve, which is determined in terms of its first and second order derivatives. The external energy is based on the positional error between two curves.

In previous studies, different algorithms related to the snake model were proposed that employed various strategies to facilitate the stabilization and faster convergence (Amini et al., 1990; Lam and Yan, 1994; Williams and Shah, 1992; Xu et al., 1994) of the associated iterative energy minimization process. To overcome the limitations of the snake model, an improved version called B-snake was proposed in Flickner et al. (1994), Menet et al. (1990), Wang et al. (1996). In B-snake approaches, a parametric B-spline representation is used rather than the standard spline-based representation, which facilitates local control of the shape of the curve by regulating related control points.

A new version of B-snake called NURBS-snake was presented in Meegama and Rajapakse (2003). The NURBS-snake provides more flexibility than the B-snake model because the weight parameters can adapt to local curvature changes. Moreover, in Cai et al. (2011), a snake-based curve deformation model was proposed to reconstruct space curves, where the necessity of matching was relaxed. However, the order of the NURBS-snake, the weights, and the knot sequence is assumed to be fixed, although this limitation was relaxed in Xie et al. (2012), where a new method was proposed that employed