Computer Aided Geometric Design 32 (2015) 5–21

Contents lists available at ScienceDirect

Computer Aided Geometric Design

www.elsevier.com/locate/cagd

Subdivision surface fitting to a dense mesh using ridges and umbilics ☆

Xinhui Ma [a,*], Simeon Keates [a], Yong Jiang [b], Jiří Kosinka [c]

[a] School of Engineering, and Computing and Applied Mathematics, Abertay University, Dundee, DD1 1HG, UK
[b] Nanjing University of Information Science and Technology, 210044, China
[c] Computer Laboratory, University of Cambridge, Cambridge CB3 0FD, UK

ARTICLE INFO

Article history:
Received 30 May 2013
Received in revised form 25 October 2014
Accepted 31 October 2014
Available online 20 November 2014

Keywords:
Subdivision surface fitting
Feature alignment
Ridges
Umbilics
Hausdorff distance
Principal curvature vector

ABSTRACT

Fitting a sparse surface to approximate vast dense data is of interest for many applications: reverse engineering, recognition and compression, etc. The present work provides an approach to fit a Loop subdivision surface to a dense triangular mesh of arbitrary topology, whilst preserving and aligning the original features. The natural ridge-joined connectivity of umbilics and ridge-crossings is used as the connectivity of the control mesh for subdivision, so that the edges follow salient features on the surface. Furthermore, the chosen features and connectivity characterise the overall shape of the original mesh, since ridges capture extreme principal curvatures and ridges start and end at umbilics. A metric of Hausdorff distance including curvature vectors is proposed and implemented in a distance transform algorithm to construct the connectivity. Ridge-colour matching is introduced as a criterion for edge flipping to improve feature alignment. Several examples are provided to demonstrate the feature-preserving capability of the proposed approach.

1. Introduction

Fitting a sparse and smooth surface that approximates dense data is a desirable goal in applications such as compression, recognition, and reverse engineering. The simplified surface models or data are obtained for convenient downstream processing, e.g. surface design, animation and manufacturing. Commonly, there are two types of surfaces which are used to fit dense data: NURBS (Non-Uniform Rational B-Spline) and subdivision surfaces. Ma et al. (2004), Lavoué et al. (2007), Panozzo et al. (2011) fit these surfaces to dense meshes. Sometimes one is interested in fitting pure geometric surfaces. For example, the first author's previous work (Ma and Cripps, 2011) fits generalised Cornu spirals to dense surface points to preserve the original shape. Most existing methods fit B-spline surfaces to 3D points with simple topological type. In the case of complex topology, it is an extremely difficult task to handle continuity conditions among neighbouring surfaces. Subdivision surfaces can easily represent arbitrary topology in a compact form and achieve the same accuracy as NURBS.

Subdivision surfaces have recently become of great interest due to their high potential in rendering and shape design. A principal achievement in shape design is that subdivision surfaces are now compatible with the industry standard NURBS thanks to the work of Cashman et al. (2009) generalising the schemes of Catmull and Clark (1978) and Doo and Sabin (1978) to general degrees. The scheme of Loop (1987) is a popular subdivision scheme for triangulations. NieBner et al. (2012)

☆ This paper has been recommended for acceptance by Kai Hormann.

* Corresponding author. Tel.: +44 138230 8224.
E-mail address: x.ma@abertay.ac.uk (X. Ma).

http://dx.doi.org/10.1016/j.cagd.2014.10.001

achieve unprecedented quality and speed in rendering by fast evaluation of subdivision surfaces on GPUs (Graphical Processing Units). Subdivision surfaces ideally suit the paradigm of hardware tessellation on GPUs. Only the vertices of a coarse model need to be animated; the GPU can amplify the coarse geometry on-the-fly with very little memory bandwidth to produce a dense tessellation of the surface. Panozzo et al. (2011) predict that subdivision-based modelling is expected to replace polygonal modelling even for real time applications.

This paper proposes a new approach to fit subdivision surfaces to dense meshes. Most existing algorithms aim at obtaining a mesh with a good quality of accuracy and regularity. However, we focus on preserving the features, since the convergence is improved by the alignment of the connectivity with salient features. The coarse control mesh is constructed from ridges, umbilics, and ridge-crossings of the original dense mesh. Ridges are geometrically and perceptually salient surface features and are used for shape recognition as said in Ohtake et al. (2004). Thirion (1996) states that these features can characterise the overall shape of the original data and are successfully used in image processing for registration. They also constitute a natural connectivity, since ridges start and end at umbilics and cross at ridge-crossings as stated in Porteous (1994). The natural connectivity matches the requirement of connectivity for a control mesh. Hence, we use these features to construct a coarse control mesh of subdivision surfaces to fit the overall shape of the original dense mesh.

The flow of the proposed approach is: First, ridges, umbilics, and ridge-crossings are extracted from the input dense mesh. Second, the significant features are filtered. Then, a connectivity of the feature points is constructed and its alignment with ridges is improved. Finally, a coarse control mesh is constructed and a subdivision surface is generated to fit the original dense mesh.

The main contributions of the approach are:

- The natural ridge-joined connectivity of umbilics and ridge-crossings is used as the connectivity of control mesh for subdivision.
- A metric of Hausdorff distance including curvature vectors is proposed and implemented in a distance transform algorithm for connectivity construction and feature alignment. An accurate arc-length estimation formula is derived to define the metric.
- A criterion of ridge-colour matching is introduced for edge flipping to improve feature alignment.

The proposed algorithm will work on noisy data as well, thanks to the noise filtering capability of the adopted feature filtering approaches (Ohtake et al., 2004; Cazals and Pouget, 2005), as discussed in Subsection 3.4. Furthermore, since the proposed method aims to preserve the salient features using as few points as possible, detailed features and noisy data will not have much effect on the fitting quality.

The paper is structured as follows. In Section 2, previous related work is reviewed. In Section 3, we discuss why we choose ridges and umbilics, and how to extract and filter such features. Construction and optimisation of connectivity are introduced in Section 4. Section 5 derives the metric of Hausdorff distance including curvature vectors. The calculation of control mesh is presented in Section 6. In Section 7, we demonstrate and evaluate the proposed method on several examples. Finally, we give concluding remarks.

2. Previous work

Subdivision surface fitting has been investigated by many researchers. Most existing methods can be classified as follows: mesh simplification, parametrisation, face clustering, feature extraction and others. Usually, a two-step process is adopted. Firstly, a coarse connectivity is constructed. The second step involves optimising the geometry and regularity of the connectivity. In this review, we will focus on methods that preserve features and alignment.

Hoppe et al. (1994) simplify a dense triangle mesh and construct a control mesh for Loop subdivision by optimising vertex positions. Lee et al. (2000) extend this approach using displacement mapping to approximate projectability during simplification. Suzuki et al. (1999) inversely refine an interactively defined initial control mesh. The control mesh is constructed using an iterative local approximation. Kanai (2001) simplifies the dense mesh using a modified QEM method. Ma et al. (2004) distinguish different types of vertices and edges during the simplification process to preserve sharp features. Panozzo et al. (2011) use fitmaps to achieve adaptive mesh simplification. Usually, following simplification, optimisation steps are used to optimise the geometry or connectivity using distance minimisation, collapsing, splitting or swapping edges of the control polyhedron. Approaches based on mesh simplification are easy to implement by using an established mesh simplification algorithm. However, they may need extensive computing time due to the large amount of data and geometric optimisation. It is also not clear how these methods align to salient features.

Parametrisation methods fit parameter equations for a selected domain and construct the coarse control mesh from these domains. Ma and Zhao (2002) interactively define a topological model for parametrisation and fit Catmull–Clark surfaces using linear least squares. Boier-Martin et al. (2004) introduce a method for computing parametrisations of triangulated manifolds over quadrilateral domains. The creation of the base domain is performed through a combination of clustering methods which control the shape and flatness of clusters. Lai et al. (2006) use feature sensitive parametrisation to allocate more parameter space to highly curved feature regions, so that more control points will be provided in those regions. Li et al. (2006) consider global parametrisation and quad domain remeshing to fit T-splines. Parametrisation methods can produce