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Anisotropic quadrangulation

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ABSTRACT

Quadrangulation methods aim to approximate surfaces by semiregular meshes with as few extraordinary vertices as possible. A number of techniques use the harmonic parameterization to keep quads close to squares, or fit parametrization gradients to align quads to features. Both types of techniques create near-isotropic quads; feature-aligned quadrangulation algorithms reduce the remeshing error by aligning isotropic quads with principal curvature directions. A complementary approach is to allow for anisotropic elements, which are well-known to have significantly better approximation quality.

In this work we present a simple and efficient technique to add curvature-dependent anisotropy to harmonic and feature-aligned parameterization and improve the approximation error of the quadrangulations. We use a metric derived from the shape operator which results in a more uniform error distribution, decreasing the error near features.

1. Introduction

Most common techniques for generating meshes from range scans and volumetric data produce irregular meshes with complex connectivity. A surface can be stored in a much more compact form, simplifying and speeding up rendering and processing if it is converted to a predominantly regular mesh, with only a small number of irregular vertices and faces. It is desirable to minimize the number of vertices in the semiregular mesh, while keeping it close to the original mesh.

Recent quadrangulation algorithms use a *global parameterization* of a mesh; the new mesh is obtained using a regular sampling pattern in the plane. Quite often, the parameterization is optimized to be as isometric possible. However, isometric parameterizations may be far from optimal for surface remeshing, if the goal is to obtain a surface as close as possible to the original for a given number of faces. For example, a cylinder can be mapped isometrically to the plane, resulting in a uniform sampling pattern on the surface. It can, however, also be meshed with single long quads stretched along the axial direction, with the same approximation error. We call quadrangulations that adapt the quad aspect ratio to the surface shape *anisotropic.* We present a *simple* and *robust* method for computing anisotropic quadrangulations with quad aspect ratios adapted to local curvature, obtaining a good surface approximation with fewer quads.

Our method utilizes a curvature-based surface metric and computes the parameterization using this metric, rather than the Euclidean metric. Our approach is compatible with most parameterization methods that only rely on intrinsic quantities and vector fields on the surface.

Defining a metric for meshes is conceptually simple: we assign a new length to each edge. However, each edge length has to satisfy local triangle inequality constraints. It is a surprisingly difficult task to ensure that no inequality is violated, and while it may still be possible to compute a parameterization, the results may not have the desired anisotropic behavior (Section 5). We solve this problem using the idea of a high-dimensional embedding (Pottmann et al., 2004; Cañas and Gortler, 2006b): the Euclidean metric in the higher-dimensional space defines the new edge lengths for the

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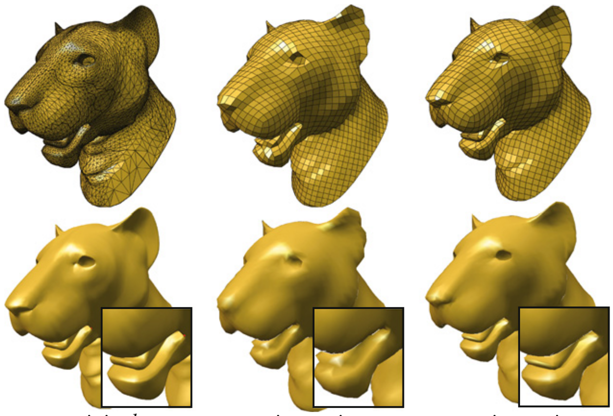

Fig. 1. Quadrangulations of a lion head model. Left: the original model; middle: isotropic feature-aligned quadrangulation (25% reduced); right: anisotropic feature-aligned quadrangulation.

mesh. The embedded vertex coordinates consist of the original positional and normal coordinates, making the new edge length computation straightforward.

2. Related work

The literature on parameterization, remeshing and quadrangulation is vast; Pottmann et al. (2004), Cañas and Gortler (2006b) and D'Azevedo and Simpson (1991) are the most closely related to our work. Our key observation is that the high-dimensional embedding proposed in Cañas and Gortler (2006b) to obtain anisotropic quadrangulations with the quad aspect ratio determined by the ratio of principal curvatures can be applied in the context of a particular class of parameterization techniques, and yields robust results while preserving fine surface features.

There are many related works considering optimal anisotropic meshes in function approximation context (some recent work includes Babenko et al., 2006, Cao, 2007 and Mirebeau, 2010). Starting from D'Azevedo and Simpson (1991), anisotropic mesh generation is often based on defining a suitable metric for the desired approximation measure, so that the isotropic triangulation in this metric results in optimal approximation. Our approach can be viewed as an application of the same general idea to the surface quadrangulation problem for a particular choice of error measure.

Many recent quadrangulation methods (in contrast to the work based on the construction of base complexes by simplification; Eck et al., 1995; Lee et al., 1998; Khodakovsky et al., 2003; Daniels et al., 2009) have similar structure: a global parameterization is obtained by solving equations for gradients of parametric functions, and a new mesh is generated by following parametric lines. The two main categories of methods of this type are harmonic and feature-aligned.

Harmonic and conformal methods (for brevity we will we refer to both as harmonic) are robust, efficient and typically produce good results even for complex meshes for a suitable choice of singularities and boundary conditions. Some quadrangulation methods use harmonic maps directly Dong et al. (2006), Tong et al. (2006). These methods can be viewed as minimizing nonconformality of the map, while allowing significant area scaling; nonlinear methods such as Sheffer and de Sturler (2001), Springborn et al. (2008) are needed to guarantee a one-to-one parameterization. Extreme area distortion is reduced by adding singularities (or "cones") to the parameterization, with several methods for automatic placement of singularities proposed in Dong et al. (2006), Ben-Chen et al. (2008) and Springborn et al. (2008). These techniques allow explicit user control over the number of irregular points on the mesh. The downside of harmonic techniques, especially in the context of remeshing, is that nonintrinsic shape information is not used directly.

The shape information can be taken into account in two distinct ways to minimize the approximation error. Locally, a smooth shape can be characterized by its shape operator. Fig. 2 shows two ways of taking the shape operator into account (with principal curvature directions scaled by inverse principal curvatures shown in red).

A "perfect" quad of a given area approximating a surface is *aligned*, i.e., has edges parallel to principal curvature directions and *anisotropic*, i.e., has aspect ratio inversely proportional to the ratio of principal curvatures. This corresponds to two classes of feature-aware parameterization techniques.

Feature-alignment methods (Ray et al., 2006; Kälberer et al., 2007; Bommes et al., 2009) adapt the parameterization to the shape by aligning new mesh elements with a feature field, typically derived from the principal curvature direction field, either by smoothing, or interpolation of salient features. The singularities of the parameterization are determined by the singularities of the field, so the feature field cannot match the actual curvature field too closely: substantial smoothing is needed to keep the number of singularities small. The shape of the quads generated by these techniques tends to be uniform, rather than anisotropic: one can view these techniques as minimizing nonisometry, while aligning with the feature field. Bommes et al. (2009) permit a degree of anisotropy, penalizing changes in length less than changes in the direction, but without relating these to curvature.