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Special section on aging and weathering

Fracture modeling in computer graphics

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ABSTRACT

While object deformation has received a lot of attention in Computer Graphics in recent years, with several good surveys that summarize the state-of-the-art in the field, a comparable comprehensive literature review is still needed for the related problem of crack and fracture modeling. In this paper we present such a review, with a special focus on the latest advances in this area, and a careful analysis of the open issues along with the avenues for further research. With this survey, we hope to provide the community not only a fresh view of the topic, but also an incentive to delve into and explore these unsolved problems further.

1. Introduction

Physically plausible object deformation and fracture have been of central importance in many fields, and particularly in Computer Graphics since more than 25 years ago [1,2]. Different areas, such as architecture and fabrication, usually require very precise simulations, for which numerical models have been devised using a combination of continuum mechanics, dynamics, differential geometry, calculus and Computer Graphics, among others. As a body can undergo many physical phenomena, fractures are essential to the movie and video game industries because of the explosions or shattering bodies required. In general, the phenomena we study in this survey can be considered as ubiquitous, as can be observed in almost every structure, from crystals to entire buildings.

In spite of its importance, the study of fractures is still a non-closed problem, with several open issues to be dealt with, most of which result from the many approximations and simplifications introduced to simulate the intrinsic complexities of this phenomenon. Advances in this field would open new frontiers for applications such as simulation and prototyping of fragile objects, resistance assessment and model resilience studies.

There are a number of good reviews on deformable models in Computer Graphics [1,3] as well as aging/weathering techniques [4] that touch on the topic of fracture processes. However, we feel that a deep review of the current state-of-the-art in crack and fracture modeling techniques is missing. Thus, in this paper we aim to fill this gap with a comprehensive review of the work done thus far, and, to improve understanding and strengthen the relationships among the different works carefully classifying them according to several criteria.

2. Overview

After an introduction to the mathematical background needed to understand the basic principles of object deformation and the phenomenon of fracture (Section 3), we present our principal classification of the different methods involved in the fracture process:

- Physically based methods (Section 4), are those that follow a simulation-based approach to compute the fracture opening, propagation and appearance. Among these, we sub-classify the state-of-the-art in the field into
  - Mass-spring models (Section 4.1), where the object is approximated by a finite set of masses, pairwise joined by springs, each with its own defining parameters.
  - Finite element methods (Section 4.2), that partition the object into a set of disjoint elements (e.g., tetrahedrons) joining at discrete points. When the problem is formulated in terms of these points, then it is converted into a set of simpler algebraic equations, which are then solved to establish the behavior of the system.
  - Meshless methods (Section 4.3), where the model is approximated with a set of unconnected calculation points that are simulated. The value for any other point in the model is obtained by interpolation.

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   - Other approaches (Section 4.4) cover those, which do not fall into the previous categories, but rather follow physical principles for their simulations of the fracture process.
- Geometry-based methods (Section 5), also known as procedural methods, seek plausible patterns but are not interested in a physically accurate phenomena description.
- Example-based methods (Section 6) try to mimic real-world fractures by copying the behavior observed in real phenomena. These methods, which build on both Computer Vision and Computer Graphics techniques, usually extract parameters from images and then apply these to generate a new fracture.

Finally, in our conclusions (Section 7), we present comparisons and further classification schemes to ensure that the reader has a comprehensive view of the most recent developments in this area. This includes Table 2, which provides further details of the main techniques reviewed in this survey. Some avenues for future research are also outlined at the end.

3. Background

In this section we will briefly introduce the main physical concepts behind generating and propagating fractures.

3.1. Stress and strain

In continuum mechanics, we define the physical quantities of an object as a continuous function in space (and time). In general, we define the rest shape of an undeformed object as the connected subspace $M \subset R^3$ [1,3,5]. Each point $\mathbf{x} \in M$ has its own properties defined at its coordinates $\mathbf{x}$ inside the object, called *material coordinates*. When we deform the object, we apply forces that move its $\mathbf{x}$ points to their new positions $\mathbf{x}'$. With the old and new positions we can define the *displacement vector field* on M as $\mathbf{u}(\mathbf{x}) = \mathbf{x}' - \mathbf{x}$, which represents the positional differences between the current point and rest positions. Refer to Fig. 1 for a graphical representation.

We usually measure deformation in terms of the so-called *strain*, which we often define as a normalized measure of the body deformation. This measure represents the displacement between particles in the body relative to a reference length. Basically, the strain measures the local deviation of a given deformation from a rigid-body deformation. As the deformation in different directions might be different, the strain is generally expressed as a *tensor*. In three dimensions, this tensor is of order 2. Given the field $\mathbf{u}(\mathbf{x})$, we can compute the *elastic strain* ε of a point at a given time,

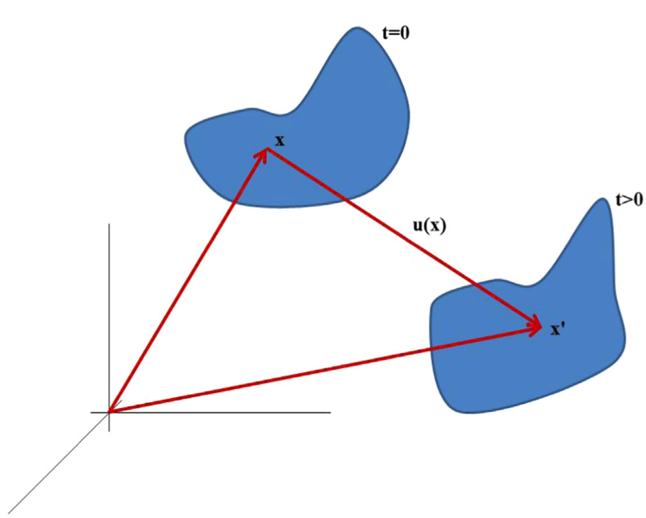

Fig. 1. The displacement vector field $\mathbf{u}(\mathbf{x})$.

simply by relating it to the gradient $\nabla\mathbf{u}$. Observe that $\nabla\mathbf{u}$ is a 3×3 matrix of the derivatives $(\nabla\mathbf{u})_{ij} = \partial_j \mathbf{u}_i$. In Computer Graphics the strain is usually defined for small deformations as one of

$$\varepsilon_G = \frac{1}{2}(\nabla\mathbf{u} + [\nabla\mathbf{u}]^T + [\nabla\mathbf{u}]^T \nabla\mathbf{u})$$
$$\varepsilon_C = \frac{1}{2}(\nabla\mathbf{u} + [\nabla\mathbf{u}]^T)$$

The first one (i.e., ε_G) is called the Green-Lagrange's strain tensor and ε_C the Cauchy strain tensor, is its linearized counterpart.

Based on the strain tensor, we can compute the *stress tensor* $\sigma \in R^{3\times3}$, which provides information about the forces acting on a point when the body is deformed. Of course, this relationship strongly depends on the properties of the material, and can be quite complex. In Computer Graphics it is customary to use Hook's law:

$$\sigma = E \cdot \varepsilon$$

where E is a rank 4 tensor which relates both tensors σ and ε in a linear way, which is useful for small deformations. On the other hand, other definitions of stress are required for large deformations, such as the Piola–Kirchhoff stress tensor, which expresses the stress relative to the reference configuration (in contrast to the Cauchy stress tensor that expresses the stress relative to the present configuration); the Biot stress tensor, which expresses the forces due to stretch only applied in the undeformed body per unit undeformed area; or the Kirchhoff stress tensor, which is widely used when there is no change in volume during plastic deformation [6]. Another possibility is the Saint Venant–Kirchhoff model:

$$\sigma = \lambda Tr(\varepsilon_G) I_3 + 2\mu\varepsilon_G$$

where λ and μ are constants specific to each material and define the way it is deformed, and I_3 is the 3×3 identity matrix. For more information on these tensors, we refer the interested reader to the works authored by Bonet and Wood [6] and Chakrabarty [7].

In general, the stress tensor σ is a symmetric 3×3 matrix, so it has 3 real eigenvalues. These eigenvalues correspond to the stresses in the principal directions, represented by its respective eigenvectors to the principal stress directions. Positive eigenvalues indicate tension, while negative eigenvalues represent compression.

It is possible to compute the body force f for each point from σ as

$$f(\mathbf{x}, t) = \nabla \cdot \sigma(\mathbf{x}, t)$$

whose elements are $f_i = \sum_j \partial_j \sigma_{ij}$. With these forces $f(\mathbf{x}, t)$, we can model deformation using the equations of motion. In general, these equations are posed in terms of the density ρ of the material, which again is a function of position $\mathbf{x}$ and time t:

$$\rho(\mathbf{x}, t)\frac{\partial^2}{\partial t^2}\mathbf{x} = f_i(\mathbf{x}, t)$$

However, for general cases it is almost impossible to find an analytical solution, and we must resort to numerical methods, which are the subject of the following sections.

3.2. Brittle and ductile fractures

We can define an *elastic* material as one that will return to its original shape when the external forces on it cease to exist. To the contrary, a *plastic* material will not go back to its original configuration. Real materials usually have a limited elastic behavior, and if deformed beyond a certain threshold (called *elastic limit* or *yield point*), they will undergo a plastic deformation. If the material is deformed further, there is another limit, called the *failure threshold* σ_{max}, which is the point at which the material s. This *failure*