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Technical Section

Fast corotational simulation for example-driven deformation ☆

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ABSTRACT

We present a fast corotational finite element framework for example-driven deformation of 3-dimensional solids. An example-driven deformation space and an example space energy is constructed by introducing the modified linear Cauchy strain with rotation compensation. During this simulation, our adopted total energy functional is quadratic, and its corresponding optimization can be quickly worked out by solving a linear system. For addressing the possible errors, we propose an effective error-correction algorithm. Some related factors including the parameters and example weights are also discussed. Various experiments are demonstrated to show that the proposed method can achieve high quality results. Moreover, our method can avoid complex non-linear optimization, and it outperforms previous methods in terms of the calculation cost and implementation efficiency. Finally, other acceleration algorithms, such as the embedding technique for handling highly detailed meshes, can be easily integrated into our framework.

1. Introduction

Simulation of deforming objects is one of the active topics in computer graphics, and it has been widely applied in the movie industry, digital entertainment and product design. Over the years, researchers have conducted a large amount of excellent work on model deformation simulation, namely, editing, material modeling, user interaction and other related issues. With the repaid development of deformable model acquisition, example-driven deformation techniques [1–3] have received much attention, and their applications have also become the extension of the key frame animation and physical simulation.

Most of the approaches in the example-driven deformation domain construct an energy functional of the object and solve the corresponding optimization problems for obtaining the node positions and reconstructing the shapes. In general, the energy functional is composed of two parts, which are the deformation energy and the constraint energy introduced by examples. Moreover, the later part influences the simulation results by constructing an example deformation space. Both parts can be constructed by geometric and physical methods. In comparison, physical methods can more easily reflect the physical properties of the model materials.

Example-driven deformation can produce more diverse effects than conventional key frame techniques in many applications. However, the induced optimization problem always leads to complex non-linear solutions. This limits the simulation response time as well as the solution scale.

In this paper, an efficient computing framework for example-driven deformation is presented. Inspired by existing approaches, we construct an energy functional of the deformable models in a physically based context. For modeling the deformation metric, we apply a revised Cauchy strain for efficient simulation. At the same, the corotational method is integrated to tackle the large deformation problem and also to form the example deformable space.

Compared with previous methods, the proposed method is based on linear strain, and the induced optimization can be worked out quickly by simply solving a linear system. To address the possible errors, an effective error-correction algorithm is proposed. Moreover, it is easy for our framework to integrate the existing speed-up algorithms such as the embedding technique for highly detailed meshes.

2. Related work

We will briefly review related work on physically based animation, shape interpolation, example driven deformation and other relevant topics.

In the 1980s, the pioneering work in the field of *physically based animation* was proposed by Terzopoulos et al. [4]. The main

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goal of this research field is to improve the simulation reality, speed and stability for computer animation. A large number of studies have been performed, including mass–spring, FEM and meshless approaches. Many issues have been thoroughly discussed, including the approximated solution of a control PDE, processing on large deformations and other topics. A more detailed overview can be found in [5–7].

To tackle the nonlinear issue in a large deformation, the *corotational method* [8], which is also called stiffness warping [9,10], has been widely applied. This category of methods factorizes the large deformation into two parts, which are the linear deformation and the rigid rotation quantities. In each simulating step, the stiffness matrix composes the linear stiffness and rotation compensation. Thus, the treatment for non-linear factors can be simplified and the simulation procedure can be sped up. Recently, the corotational technique has become a mainstream method in many computer graphics applications [11,12].

Shape interpolation is an important issue in geometry processing. Many techniques have been proposed for morphing and shape editing. Alexa et al. [13,14] proposed an array of methods for shape interpolation as rigid as possible. Subsequently, many researchers have conducted a substantial amount of work addressing the rotation issue during interpolation, such as Lipman et al. [15]. Based on the differential coordinates of a surface mesh, Xu et al. [16] proposed a Poisson equation based formulation for shape interpolation. Huang et al. [12] added dynamic effects in the shape interpolation. There is a substantial amount of other work, such as [17,18] which tackled the interpolation issue of spatial rotation. In addition, as a natural extension of shape interpolation, Kilian et al. [19] discussed techniques to construct deformation space from examples.

Surface deformation has received continuous attention for many years [20]. The MeshIK [21] is one of the representative methods for creating deformable models based on deformation examples. This method has been extended in later publications [22,1]. Moreover, it is one of the research bases for many techniques of example driven deformation.

Frohlich et al. [1] proposed a framework of *example-driven deformation* based on discrete shells, and the framework can generate static deformation of triangle meshes, combining examples and physical laws. More recently, Martin et al. [2] presented an example-based method for elastic materials that implements dynamic deformations following the physical laws. They adopted nonlinear Green strain to create a deformation metric space and construct the solved energy functional, including the elastic deformation energy and the energy that reflects the examples effects. Then, they solved and obtained the deformable shapes of the elastic models by using the Newton–Raphson optimization, and a performance nickel is the nonlinear optimization. Later, Schumacher et al. [3] implemented a similar method for elastic–plastic deformation. Specifically, as an extension of the method in [3], Coros et al. [23] presented a method for controlling the motions of active deformable characters. Koyama et al. [24] also formulated an analogous concept to generate plausible animation in a shape-matching framework.

In this paper, we present an example-driven deformation framework, in which the enhanced Cauchy strain is adopted to quantize the deformation metric and a corotational technique is applied to compensate the artifact induced by linear calculation in large deformations. In the following sections, the formulation of the problem is presented and the applications of static and dynamic deformation are discussed. At the same time, an effective error-correction algorithm is given. Then, we use an embedding technique to speed up the computing procedure. Moreover, several examples are presented to prove the efficiency of the method, and several related factors in deformation calculating is briefly discussed. Finally, the limitations of the proposed method and our future work are described.

3. Formulation

Without loss of generality, we assume that a given solid $\Omega \subset \mathbb{R}^3$ is discretized into a linear tetrahedral mesh with n nodes and m elements. Let $\mathbf{X}$, $\mathbf{x} \in \mathbb{R}^{3n}$ denote position vectors that describe the initial and deformed configurations, respectively. $\mathbf{x}_1, \ldots, \mathbf{x}_k \in \mathbb{R}^{3n}$ represent the position vectors of k input deformation examples. We will construct an example driven-deformation space, and define the solved energy functional by using the corotational Cauchy strain.

3.1. Corotational Cauchy strain

The behavior of deformable solids is mainly modeled as the movements of the inner points in elastic mechanics. Given a solid, the movement of any point $p \in \Omega$ can be denoted as

$$p : \Omega \times \mathbb{R} \to \mathbb{R}^3 : (\mathbf{X}, 0) \mapsto \mathbf{x}(\mathbf{X}, t).$$

In a linear FEM simulation, the procedure of solid deformation is basically depicted by the *deformation gradient*, which can be calculated by

$$\mathbf{F} = \partial \mathbf{x} / \partial \mathbf{X},$$

where $\mathbf{F}$ is not rotation invariant. In a large deformation, a rigid rotation can lead to the changing of $\mathbf{F}$ and a straightforward linear FEM calculation can produce undesirable artifacts due to the non-linear nature of the rotation transform. Thus, we apply corotational FEM simulation for the rotation compensation.

For any one tetrahedron element j, the *corotational deformation gradient* is calculated as

$$\hat{\mathbf{F}}_j^e = \partial(\mathbf{R}_j^{e\top} \mathbf{x}_j^e)/\partial \mathbf{X} = \mathbf{R}_j^{e\top} \mathbf{F}_j^e$$

where superscript e indicates that the vector or matrix is calculated from an element. $\mathbf{R}_j^e$ is a block diagonal matrix, each of whose sub-blocks is a mapping element rotation matrix. The element rotation matrix is a 3×3 unitary matrix that can be computed by the polar decomposition of the deformation gradient $\mathbf{F}_j^e$. Thus, we have a corotational version of the Cauchy strain metric for the element

$$\varepsilon_j = (\hat{\mathbf{F}}_j^\top + \hat{\mathbf{F}}_j)/2,$$

which is rotation invariant. Moreover, the *corotational Cauchy strain* of an element can be represented as

$$\boldsymbol{\varepsilon}_j^e = \mathbf{B}_j^e(\mathbf{R}_j^{e\top} \mathbf{x}_j^e - \mathbf{X}_j^e) \tag{1}$$

where $\mathbf{X}_j^e = [\mathbf{X}_{j,1}^{e\top} | \mathbf{X}_{j,2}^{e\top} | \mathbf{X}_{j,3}^{e\top} | \mathbf{X}_{j,4}^{e\top}]^\top$ and $\mathbf{x}_j^e = [\mathbf{x}_{j,1}^{e\top} | \mathbf{x}_{j,2}^{e\top} | \mathbf{x}_{j,3}^{e\top} | \mathbf{x}_{j,4}^{e\top}]^\top$ are the initial and deformed coordinates of the nodes in the j-th element, respectively. $\mathbf{B}_j^e$ is a constant matrix related to the geometry of the element, and the details can be found in [25–27]. Furthermore, a 3×3 strain matrix can be denoted as a 6-dimensional vector by using *Voigt* notation. For any one shape of the given deformable model, a unique element strain vector is defined

$$\boldsymbol{\varepsilon} = [\boldsymbol{\varepsilon}_1^e \boldsymbol{\varepsilon}_2^e \cdots \boldsymbol{\varepsilon}_m^e] \subset \mathbf{R}^{6m}$$

To quantify the deformation, we introduce $U_{deform}(\mathbf{x})$ to measure the deformation energy between the current shape $\mathbf{x}$ and the initial configuration $\mathbf{X}$:

$$U_{deform} = \frac{1}{2}\int_V \boldsymbol{\varepsilon}^\top \mathbf{D} \varepsilon \, dV = \frac{1}{2}\sum_{j=1}^{m} \boldsymbol{\varepsilon}_j^{e\top} \mathbf{D} \varepsilon_j^e V_j^e$$

where $\mathbf{D}$ is the properties matrix of the material, $\boldsymbol{\varepsilon}_j^e$ denotes the strain of the j-th element, and V_j^e is the corresponding element volume.