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Chaos and Graphics

Harmonograms

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Abstract

Harmonograms are visual designs based on harmonic analyses of strings of characters. Fourier descriptors are synthesized according to character positions and values, and then traditional methods are used to generate the resulting curves. Undersampling introduces visual artifacts that actually enhance the final designs. While their primary use is for the creation of artistic designs, harmonograms may have a practical application as a method for encoding and decoding information via the frequency domain.

Keywords: Fourier descriptor; Harmonic series; Harmonogram; Undersampling; Art

1. Introduction

Just as a *monogram* consists of "two or more letters, especially a person's initials, interwoven as a device" [1], a harmonic monogram or *harmonogram* is a design described by a harmonic series synthesized from a string of characters. The main difference between the two is that a monogram is based upon the component letter shapes, while a harmonogram is based upon the component letter positions and values.

Harmonograms should not be confused with harmonographs, which are physical devices for drawing curves using pens attached to oscillating pendula [2]. Harmonographs, invented in the 19th century, typically draw simple harmonic curves such as lissajous figures until friction brings them to a halt.

Fourier descriptors provide a convenient method for generating harmonograms. The following sections briefly introduce the principles behind Fourier descriptors, then go on to demonstrate their use in the synthetic generation of curves.

2. Fourier descriptors

Fourier descriptors are an invariant measurement for describing plane closed curves and distinguishing between different shapes. They were first suggested by R. Cosgriff in 1960, but the 1972 paper "Fourier Descriptors for Plane Closed Curves" by Zahn and Roskies [3] is generally regarded to be the seminal work in this area. In the original formulation, curves are represented as a function of arc length by the accumulated change in angle since their starting point, using a Fourier series.

For example, Fig. 1 shows how a circle can be represented by its accumulated change in angle. A starting point is chosen and its direction noted. The graph on the right is an *angular profile* that shows the current angle relative to the starting angle as the curve is traversed (dotted line). Note that the angular profile starts at $\theta = 0$ and finishes at $\theta = -2\pi$, as the total angular bend along a closed curve's length must be -2π if it is followed in the clockwise direction. In other words, following the curve will eventually bring you full circle. The solid line shows the curve's *normalized angular profile*, which is given by adding the current position along the curve's length (that is, the fraction of the curve that has been traversed) to the total angular bend. The normalized angular profile starts and ends at $\theta = 0$, hence is periodic.

Fig. 2 shows the angular profiles and normalized angular profiles of two other closed plane curves, by way of further example. In both cases the angular profiles again start at $\theta = 0$ and finish at $\theta = -2\pi$, and the normalized angular profiles again start and end at $\theta = 0$. The rightmost profile contains plateaus corresponding to linear segments of the curve and discontinuities corresponding to corners.

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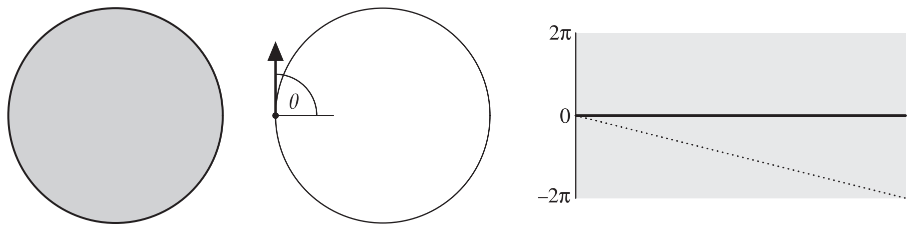

Fig. 1. A circle, a starting point on its boundary, and its angular profile.

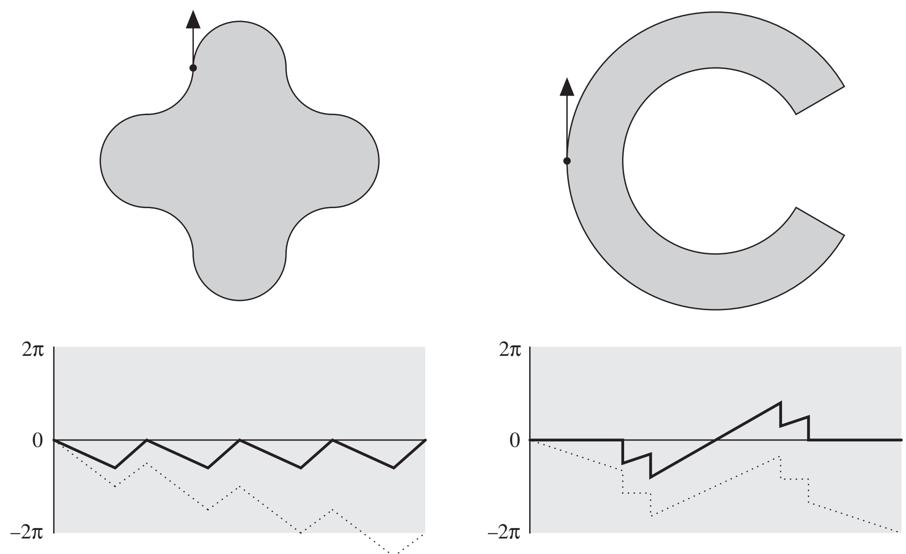

Fig. 2. Two closed planar shapes and their angular profiles.

A closed plane curve of total length L may be parameterized by its arc length l where $0 \leqslant l \leqslant L$. A cumulative angular function $\phi(l)$ may then be defined which gives the net amount of angular bend at point l relative to the starting angle. The start and end values of this function are

$$\phi(0) = 0, \quad (1)$$

$$\phi(L) = -2\pi. \quad (2)$$

This cumulative angular bend function can be normalized to the time domain corresponding to the curve's lifetime $t = [0..1]$ as follows:

$$\phi^*(t) = \phi\left(\frac{Lt}{2\pi}\right) + t. \quad (3)$$

This allows all curves with identical shape and starting point to be uniquely defined by the periodic function $\phi^*(t)$. Curves have *identical shape* if they differ only by a combination of translation, scale and rotation [3].

Note that the normalized cumulative angular bend function $\phi^*(t)$ will be always be zero if the curve is a circle, since the circle's angular bend changes constantly along its length. The function $\phi^*(t)$ therefore measures the way in which a curve differs from a circle.

The normalized cumulative angular bend function can be expanded into the frequency domain as a Fourier series. Eq. (4) shows the general form and Eq. (5) the more convenient polar form:

$$\phi^*(t) = \mu_0 + \sum_{k=1}^{\infty}(a_k \cos kt + b_k \sin kt), \quad (4)$$

$$\phi^*(t) = \mu_0 + \sum_{k=1}^{\infty} A_k \cos(kt - \alpha_k). \quad (5)$$

The curve is thus described by the Fourier descriptors A and α, where A_k is the harmonic amplitude at the kth frequency, and α_k is the phase angle at the kth frequency.

Fourier descriptors have become a standard tool for practitioners of computer vision and pattern recognition. Various approaches now exist, based on the same basic principles but differing in detail and implementation. For example, the elliptic Fourier features of Kuhl and Giardina [4] are now widely used for classifying and comparing shapes.

3. Curve reconstruction

A curve's Fourier descriptors essentially encode its angular profile as a harmonic series, from which the curve can be readily reconstructed. Fig. 3 (left) shows how sample points may be obtained at regular intervals along the curve's path by finding the accumulated angle at each