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Special Section

Particle-based shallow water simulation for irregular and sparse simulation domains

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A B S T R A C T

We propose a shallow water simulation using a Lagrangian technique. Smoothed particle hydrodynamics are used to solve the shallow water equation, so we avoid discretization of the entire simulation domain and easily handle sparse and irregular simulation domains. In the context of shallow water equations, much less attention has been paid to Lagrangian simulation methods than to Eulerian methods. Therefore, many problems remained unsolved, which prevents the practical use of Lagrangian shallow water simulations in computer graphics. We concentrate on several issues associated with the simulation. First, we increase the accuracy of the smoothed particle hydrodynamics approximation by applying a correction to the kernel function that is used in the simulation. Second, we introduce a novel boundary handling algorithm that can handle arbitrary boundary domains; even irregular and complicated boundaries do not pose a problem and introduce only small computational overhead. Third, with the increased accuracy, we use the fluid height to generate a flat fluid surface. All the proposed methods can easily be integrated into the smoothed particle hydrodynamics framework.

1. Introduction

Computational fluid dynamics has been a popular area of research in computer graphics for many years. The most work has been performed in the field of 3-dimensional fluid simulations. A 3-dimensional solution is usually required because all the fluid dynamics can be captured. Although the topic of 3-dimensional fluid simulations has been studied extensively and much progress has been achieved in the quality and speed of simulations in recent years, it remains out of the realm of real-time simulations.

Although simulating fluid dynamics is a computationally demanding problem, the real-time simulation of fluid dynamics is often desired, especially in applications where interactive behavior is required. To achieve a sufficiently fast simulation, a method that is simpler than current 3-dimensional methods must be employed, or the methods that solve the 3-dimensional fluid dynamics must be simplified. The simplification is done either by reducing the dimensionality of the problem or by simplifying the governing equation for at least part of the simulation domain. One of the most widely used approaches is the use of shallow water equations (SWE), which are a simplified version of the Navier–Stokes equations for an inviscid 2D fluid flow. The use of SWE accelerates the simulation in two ways: they simplify the governing equation, and the fluid is represented by a height field.

The SWE are usually solved in an Eulerian manner using a grid. Such solutions are well studied in the field of computer graphics but are not suitable for irregular domains that are not aligned with the grid. Much less attention has been paid to Lagrangian solutions of the SWE. In this paper, we propose a Lagrangian shallow water simulation to avoid the discretization of the entire simulation domain and to more easily handle complicated boundary domains. We utilize smoothed particle hydrodynamics (SPH) to solve the SWE. The use of SPH poses several new problems that need to be addressed:

- The use of SPH results in inaccurate evaluation of quantities, especially in areas near boundaries, because of particle deficiencies. We increase the accuracy by applying a correction to the kernel functions that are used in the simulation.
- The existing methods for boundary handling are difficult to set up correctly. Their use in the handling of irregular boundaries is also problematic because of large computational demands. We propose a novel boundary handling technique for the SPH-based shallow water simulation that can handle boundary domains with arbitrary complexity.
- Achieving a flat fluid surface is problematic because of irregular fluid sampling. We propose a new fluid generation method that produces a flat fluid surface using the fluid height.

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2. Related work

Several authors have proposed different methods for height field fluid simulations. Some of these solutions are not physically based, like that of O'Brien and Hodgins [1], where columns were used to represent the water, and the fluid flow was represented by the volume exchange between the columns. Yuksel et al. [2] proposed using wave particles to simulate the fluid surface in real time. Fluid was procedurally generated by Fournier and Reeves [3], Hinsinger et al. [4] and Belyaev [5]. Lee et al. [6] performed a real-time simulation using linear convolution on a height field.

Several physically based solutions have been proposed as well. Wave equations were used in a physically based simulation by Cords [7]. A simulation based on the SWE was first introduced to computer graphics by Kass and Miller [8]. A height field representation with an implicit integration on a uniform grid has commonly been used to solve the equations. A parallelized version of a shallow water simulation was later implemented on a GPU by Hagen et al. [9].

Many articles address the limitations of height field simulations. The following extensions to the shallow water simulation have been proposed. Sheets of fluid were added to the simulation by Thurey et al. [10] to handle breaking waves, and a particle-based bubble and foam model was added to the shallow water simulation by Thurey et al. [11]. A two-dimensional SWE simulation was coupled with a full three-dimensional simulation by Thurey et al. [12]. A hybrid method that combines shallow water simulation with uncoupled particles was presented by Chentanez and Müller [13], who also introduced non-reflecting boundary conditions that are suitable for simulating open scenes. Later, the concept of tall cells was used to achieve a real-time simulation by Chentanez and Müller [14]. Recently, Chentanez et al. [15] successfully coupled the SWE with a 3-dimensional SPH simulation.

The Lagrangian shallow water simulation was introduced to computer graphics by Lee and Han [16], who altered the standard SPH [17] to handle a height field simulation. The shallow water simulation was further enhanced by Solenthaler et al. [18], who achieved two-way coupling between the fluid and rigid bodies as well as a surface representation that reduces bumpiness and produces a flat fluid surface.

Handling of arbitrary domain boundaries was solved by Solenthaler et al. [18], but repulsion forces were used to handle steep terrain slopes. The repulsion forces are difficult to set up and compute. Boundary domains were handled by Vacondio et al. [19,20] by employing virtual particles. Fluid particles are mirrored to sample the boundary with particles. Although irregular domains can be handled, the method is computationally expensive because the virtual particles have to be generated at every time step.

3. Simulation background

Our approach to solving the shallow water equation is based on a particle-based SPH method from [16]. To compute a quantity at a position $\mathbf{p}$ within the simulation domain, SPH sums over the neighboring particles within a radius r using the kernel $W(\mathbf{p}-\mathbf{p}_j, r)$. The kernel specifies the influence of the particles depending on the distance $\mathbf{p}-\mathbf{p}_j$ between particle j and position $\mathbf{p}$. To simplify the notation, W_{ij} is used instead of $W(\mathbf{p}_i-\mathbf{p}_j, r)$ in the remainder of this paper. A quantity Q can thus by computed using SPH as

$$Q_i = \sum_j \frac{m_j}{\rho_j} Q_j W_{ij}, \tag{1}$$

where m is the mass of the particle, and ρ is the density of the particle.

The formulation of the SPH summation from Eq. (1) has to be altered to solve shallow water equations. The constant mass and the density of the particles are changed to a constant volume V and height h of the particles, respectively. This alteration leads to an evaluation of the quantity Q

$$Q_i = \sum_j \frac{V_j}{h_j} Q_j W_{ij}. \tag{2}$$

Using Eq. (2), the SWE are approximated, which leads to the evaluation of the fluid height and pressure forces

$$h_i = \sum_j V_j W_{ij}, \tag{3}$$

$$\mathbf{f}_i^{pressure} = -g\nabla h = -g\sum_j V_j \nabla W_{ij}, \tag{4}$$

where g is the gravity. The particles only move in the horizontal direction, and h_i is a property of particle i that represents the fluid height at the position of particle i as depicted in Fig. 2.

Although the shallow water equations describe inviscid flow, the viscosity is used in the simulation to minimize the numerical oscillations and to increase stability. In addition to the viscosity from [16], we symmetrize the viscous forces using the formulation

$$\mathbf{f}_i^{viscosity} = \mu \sum_j V_j \frac{\mathbf{u}_j - \mathbf{u}_i}{h_j} \nabla^2 W_{ij}, \tag{5}$$

where μ is the viscosity coefficient, and $\mathbf{u}$ is the velocity of the particle.

4. Increasing accuracy

Although a realistic fluid motion can be achieved using the equations from Section 3, oscillations in the height field may occur. High errors arise from the height computation near the boundaries because of the particle deficiency. If a part of the particle neighborhood is not filled with particles the summation is done over less particles and thus lower fluid heights are gained using Eq. (3). Similar problems might be observed in the simulation of Navier–Stokes equations where the accuracy of the density field is improved by Shepard filter (e.g. by Panizzo [21] or Akinci et al. [22]). The kernel W_{ij}^{3d} used in the computation of density is normalized and changed to

$$\tilde{W}_{ij}^{3D} = \frac{W_{ij}^{3D}}{\sum_j W_{ij}^{3D} \frac{m_j}{\rho_j}}. \tag{6}$$

Such solution can be exploited in the shallow water simulation as well. The kernel used in Eq. (3) is changed by applying Shepard filter to

$$\tilde{W}_{ij} = \frac{W_{ij}}{\sum_j \frac{V_j}{h_j} W_{ij}}. \tag{7}$$

Similar errors arise in the computation of the gradient. Unfortunately, the Shepard filter cannot be applied to the computation of the gradient; instead, an alternative approach that involves computation and inversion of a matrix must be used. This approach is not usually considered in computer graphics because it is computationally expensive for 3D simulations. Because of the 2D nature of the shallow water simulation, we were able to use this method with acceptable computational overhead.

We used a corrected kernel $\overline{W}_{ij}$ in the computation of Eq. (4)

$$\overline{W}_{ij} = M_i^{-1} \nabla W_{ij}, \tag{8}$$

$$M_i = \sum_j \frac{V_j}{h_j} \nabla W_{ij} \otimes (\mathbf{p}_i - \mathbf{p}_j), \tag{9}$$