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SCIENCE DIRECT®
Computers & Graphics 30 (2006) 46–53
COMPUTERS & GRAPHICS
www.elsevier.com/locate/cag

Supercover model, digital straight line recognition and curve reconstruction on the irregular isothetic grids

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Abstract

On the classical discrete grid, the analysis of digital straight lines (DSL for short) has been intensively studied for nearly half a century. In this article, we are interested in a discrete geometry on irregular grids. More precisely, our goal is to define geometrical properties on irregular isothetic grids that are tilings of the Euclidean plane with different sized axis parallel rectangles. On these irregular isothetic grids, we define digital straight lines with recognition algorithms and a process to reconstruct an invertible polygonal representation of an irregular discrete curve.

1. Introduction

When a straight line is digitized on a square grid, we obtain a sequence of grid points defining a digital straight-line segment. This computer representation of such a simple Euclidean object has drawn considerable attention in many applications (drawing [1], shape characterization [2–4], ...). The structure of DSL is now well known and links have been illustrated between DSL and objects from number theory or theory of words (see [5] for a survey on digital straightness). Beyond this characterization, an important task in computer vision consists in the recognition of DSL segments. More precisely, given a set of pixels, we have to decide if there exists a DSL segment that contains the given pixels. Many efficient algorithms exist to implement such a recognition process [6–9]. Based on a digital straight line recognition algorithm, we can also define a segmentation process that decomposes a discrete curve into maximal DSL segments. The next step of the segmentation process is to reconstruct a polygonal curve from the discrete data such that its digitization is equal to the original discrete curve. This process is called an *invertible reconstruction* of a discrete curve [10–12]. The invertible property is an important one in discrete geometry since it allows to convert discrete data to Euclidean ones such that no information is added nor lost.

In this article, we are interested in defining a geometry on irregular isothetic grids. More precisely, we consider grids defined by a tiling of the plane using axis parallel rectangles. Such a grid model includes, for example, the classical discrete grid, the elongated grids [13] and the quadtree based grids [14]. In [15], a general framework has been proposed that defines elementary objects and a digitization framework, the *supercover model*. An important aspect of this general framework is the consistency with classical definitions if the discrete space is considered.

Many applications may benefit from these developments. For example, we can cite the analysis of quadtree compressed shapes, or the use of geometrical properties in objects represented by interval or affine arithmetics (see discussion in [15]). Based on this irregular model, we

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0097-8493/$ - see front matter
doi:10.1016/j.cag.2005.10.009

define digital straight lines with recognition algorithms and a process to reconstruct an invertible polygonal representation of an irregular discrete curve.

Section 2 presents more formal definitions in the irregular grids: adjacency relations, objects, arcs, curves and the supercover model. Based on a definition of the irregular isothetic digital straight lines, we present algorithms to recognize maximal irregular discrete straight segments and to reconstruct invertible polygonal arcs and curves (Section 3). Experiments and results are shown in Section 4.

2. Preliminary definitions

2.1. The irregular isothetic model

First of all, we define an *irregular isothetic grid*, denoted $\mathbb{I}$, as a tiling of the plane with isothetic rectangles. In this framework, the rectangles have not necessarily the same size but we can note that the classical digital space is a particular irregular isothetic grid. In that case, all squares are centered in $\mathbb{Z}^2$ points and have a border size equal to 1. Fig. 1 illustrates some examples of irregular isothetic grids. A rectangle of an isothetic grid is called a *pixel*. Each pixel P is defined by its center $(x_P, y_P) \in \mathbb{R}^2$ and a size $(l_P^x, l_P^y) \in \mathbb{R}^2$. Before we introduce objects and straight lines in such grids, we need adjacency relations between pixels.

Definition 1 (*ve-adjacency, e-adjacency*). Let P and Q be two pixels. P and Q are *ve-adjacent* if:

$$|x_P - x_Q| = \frac{l_P^x + l_Q^x}{2} \quad \text{and} \quad |y_P - y_Q| \leqslant \frac{l_P^y + l_Q^y}{2},$$

or

$$|y_P - y_Q| = \frac{l_P^y + l_Q^y}{2} \quad \text{and} \quad |x_P - x_Q| \leqslant \frac{l_P^x + l_Q^x}{2}.$$

P and Q are *e-adjacent* if we consider an exclusive "or" and strict inequalities in the above *ve-adjacent* definition.

In the following definitions, we use the notation *k-adjacency* in order to express either the *ve-adjacency* or the *e-adjacency*. Using these adjacency definitions, several basic objects can be defined:

Definition 2 (*k-path*). Let us consider a set of pixels $\mathscr{E} = \{P_i, i \in \{1, \dots, n\}\}$ and a relation of k-adjacency. $\mathscr{E}$ is a k-path if and only if for each element P_i of $\mathscr{E}$, P_i is k-adjacent to P_{i-1}.

Definition 3 (*k-object*). Let $\mathscr{E}$ be a set of pixels, $\mathscr{E}$ is a k-object if and only if for each couple of pixels (P, Q) belonging to $\mathscr{E} \times \mathscr{E}$, there exists a k-path between P and Q in $\mathscr{E}$.

Definition 4 (*k-arc*). Let $\mathscr{E}$ be a set of pixels, $\mathscr{E}$ is a k-arc if and only if for each the element of $\mathscr{E} = \{P_i, i \in \{1, \dots, n\}\}$, P_i has exactly two k-adjacent pixels, except P_1 and P_n which are called the extremities of the k-arc.

Definition 5 (*k-curve*). Let $\mathscr{E}$ be a set of pixels, $\mathscr{E}$ is a k-curve if and only if $\mathscr{E}$ is a k-arc and $P_1 = P_n$.

If we consider pixels such that $l_P^x = l_P^y = 1$ and $(x_P, y_P) \in \mathbb{Z}^2$ (i.e. a 2D digital space), all these definitions coincide with the classical ones [16,17]. More precisely, the *ve*-adjacency (resp. *e*-adjacency) is exactly the 8-adjacency (resp. the 4-adjacency). In the following, we only consider geometrical properties of such objects. A complete topological analysis of k-curves and k-objects is not addressed here.

2.2. Supercover model on the irregular isothetic grids

Before defining the digital straight lines on the irregular isothetic grids, we have to consider a digitization model. In the following, we choose to extend the supercover model. This model was first introduced by Cohen-Or and Kaufman in [18] on the classical discrete grid and then widely used since it provides an analytical characterization of basic supercover objects (e.g. lines, planes, 3D polygons,...) [19,20].

Definition 6 (*Supercover on irregular isothetic grids*). Let F be an Euclidean object in $\mathbb{R}^2$. The supercover $\mathbb{S}(F)$ is defined on an irregular isothetic

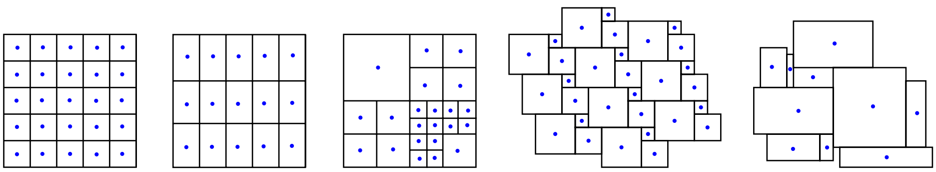

Fig. 1. Examples of irregular isothetic grids: (*from left to right*) the classical discrete grid ($(x_P, y_P) \in \mathbb{Z}^2$ and $l_P^x = l_P^y = 1$), an elongated grid ($l_P^x = \lambda$, $l_P^y = \mu$ and $(x_P, y_P) = (\lambda i, \mu j)$ with $(i,j) \in \mathbb{Z}^2$), a quadtree decomposition (for a cell of level k, $(x_P, y_P) = (m/2^k, n/2^k)$ and $l_P^x = l_P^y = 1/2^{k-1}$ for some $m, n \in \mathbb{Z}$); a unilateral and equitransitive tiling by squares: the size of the biggest square is equal to the sum of the two other square sizes; finally a general irregular isothetic grid.