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SCIENCE DIRECT
Computers & Graphics 30 (2006) 54–61
COMPUTERS & GRAPHICS
www.elsevier.com/locate/cag

A reversible and statistical method for discrete surfaces smoothing

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Abstract

In this article we propose an original reversible method for discrete surface smoothing. This method is based on a statistical estimation of the discrete tangent plane on the voxels of the discrete surface. A geometrical constraint is used to control the recognition of the tangent plane. The resulting surface representation allows us to get both smooth normal vectors of the surface and a smooth surface mesh while preserving the geometrical properties of the surface.

Keywords: Digital surfaces; Smoothing surface mesh; Euclidean nets; Discrete normals; Visualization; Smoothing

1. Introduction

Processing data sets of three-dimensional discrete images brings up the problem of extraction and representation of geometric features, and of the visualization of the surface of 3D objects. The initial volume object can be visualized as a set of 6-connected voxels (also called cuberille representation) [1]. But this representation in the discrete space is neither convenient for the analysis of geometric properties of the object nor for the visualization.

A polygonal representation of the boundary of a discrete object is usually used to represent its surface and to perform rendering. One of the first approaches to obtain such a representation was the marching cube algorithm [2]. This method has several drawbacks both from the geometrical and the topological points of view. Other algorithms exist which associate a surface mesh to a discrete surface. For example Türmer and Wütrich triangulate the surface by associating centers of voxels to each other [3]. Since the direct rendering of the surface obtained after such a triangulation is not smooth, normal vectors are computed in discrete space using a varying neighborhood size [4,5]. Then the surface is rendered using Gouraud shading [6]. This rendering technique gives good results, but it smooths only the normal vector of the discrete surface and not the geometry of the surface net. Other methods use deformable models to extract a continuous surface from the original discrete surface [7,8].

An alternative consist in smoothing the object surface by moving the points of the discrete surface. In [9], Braquelaire and Pousset define Euclidean nets as a 3D extension of the model of Euclidean paths [10,11]. In this model, each surface point may be moved inside the unit cube containing it. The smoothing is thus reversible and the original surface can be retrieved from the smoothed one. In the proposed method the points of the discrete

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0097-8493/$ - see front matter
doi:10.1016/j.cag.2005.10.024

surface were moved according to a projection onto some discrete tangent planes. This plane was estimated by searching for local geometric configurations of voxels called tricubes [12,13].

The main drawback of this method is the small neighborhood size which is used to determine the discrete tangent plane. Therefore, the precision of the final result is limited to a local analysis of the discrete surface. In recent works [14], Coeurjolly suggests to use a statistical computation of the discrete tangent plane to obtain the normal vectors of the discrete surface.

In this work we develop this approach and propose a statistical method to recognize accurate tangent plane with a varying neighborhood size. We then use this method to enhance the construction of a smoothed Euclidean net associated with a discrete surface. This method permits to obtain both smooth normal vectors and smooth surface mesh.

In the following section, we recall some definitions about Euclidean nets. Then, in Section 3, we introduce the statistical estimation of the discrete tangent plane. In Section 4, we show how to apply a geometric constraint to control the recognition of the discrete tangent plane. Section 5 addresses the problem of transforming the surface from discrete space to a new surface net in continuous space. Afterward, in Section 6 experimental results on real data are presented. Finally, we conclude by future work and implication of this work.

2. Discrete and Euclidean nets

Let us now give some basic definitions used in the following. The coordinates of a point P are denoted by the tuple (x_P, y_P, z_P). The set $\mathbb{Z}^3$ of points with integer coordinates is called the *discrete space* and its elements are called *discrete points*. We denote by upper case letters the points of the discrete space $\mathbb{Z}^3$ and by lower case letters the points of the Euclidean space $\mathbb{R}^3$.

Definition 1 (*Andrès [15]*). A discrete naive plane $\mathscr{P}(a, b, c, \mu, \omega)$ is the set of points (x, y, z) of $\mathbb{Z}^3$ satisfying the double inequality $\mu \leqslant ax + by + cz < \mu + \omega$, with a,b,c,$\mu$, $\omega \in \mathbb{Z}$ and $\omega = max(|a|, |b|, |c|)$.

The vector of coordinates (a, b, c) is the normal vector of the discrete plane. The coefficient μ describes the position of the discrete plane and ω corresponds to the thickness. In the same way, a discrete naive line $\mathscr{D}(a, b, \mu, \omega)$ is characterized by the normal vector (a, b), by the position in the plane (μ) and by the thickness $\omega = max(|a|, |b|)$ [16].

A *voxel* is a colored unit cube, the center of which is a discrete point. The coordinates of a voxel are the coordinates of its center. An *image* is a set of voxels and the *image domain* is the set of voxel centers. In the following we consider images for which the domain is a parallelepipedic set of discrete points. The voxel based approach has been chosen to define the boundary of an object. More precisely, the *discrete surface* of an object V is the subset $S(V)$ of V such that each voxel of $S(V)$ is 6-connected to at least a voxel of the complement of V in the image. A voxel of $S(V)$ is called a *surface voxel* of V.

A surfel is defined as the intersection of two 6-adjacent voxels. In the same way, a linel is defined as the intersection between two 4-adjacent pixels one of which belongs to the discrete line. From these definitions surfels and linels are differentiated according to their configurations. Fig. 1 illustrates different types of surfels and linels. Three different types of surfels are illustrated in the image (b) and the other surfels of type 4,5 and 6 are defined according to the opposite direction from respectively the orientation of surfels 1, 2 and 3.

The following definitions are extensions for the three dimensional case of the definitions used in the model of Euclidean paths [11].

Definition 2. Let $P = (x_P, y_P, z_P)$ be a discrete point. The *cell* of P is the set of points p of $\mathbb{R}^3$ verifying: $|x_P - x_p| < \frac{1}{2}$, $|y_P - y_p| < \frac{1}{2}$, $|z_P - z_p| < \frac{1}{2}$.

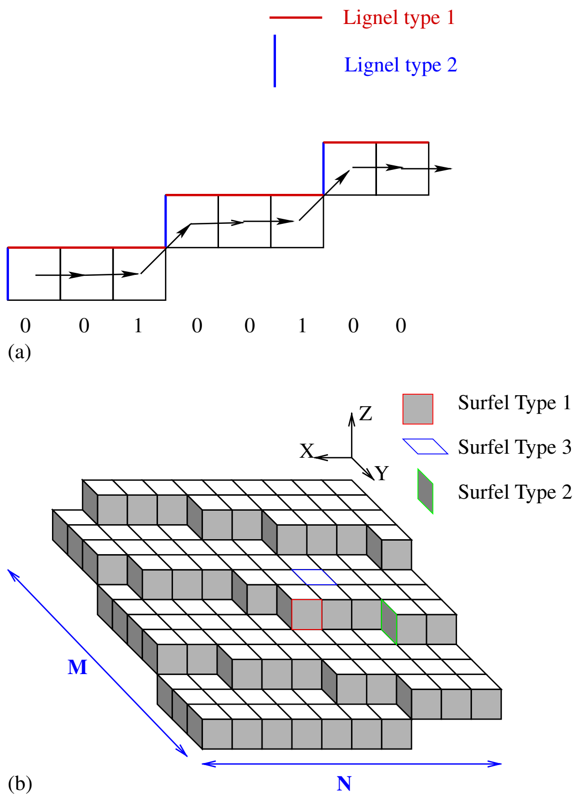

Fig. 1. Illustration of the different type of linels (a) and surfels (b).