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Fractal board games

Cameron Browne

SWiSHzone.com Pty Ltd, The Basement, 33 Ewell St, Balmain 2041, Australia

Abstract

Connection games are a recent genre of abstract board games with some interesting geometrical properties. We introduce a recursive metarule with which existing connection games can be expanded into fractal-like variants with recursively defined play. Computer graphics are used to extrapolate a theoretical game played on a continuous potential field to a similar set of games that are truly fractal in nature.

Keywords: Fractal; Abstract board game; Connection game; Hex; Y

1. Connection games

Connection games are a family of abstract board games in which players strive to complete a particular type of connection with their pieces. The inherent nature of connection allows these games to be quite complex despite having extremely simple rules. The basic concept of connection will be familiar to most players, making these games intuitive and easy to grasp though usually hard to play well.

Fig. 1 shows two famous connection games, Hex and Y, and a recent one called Quax. Each game is played on a different board, but their rules are similar and very simple; players take turns placing a piece of their color on an empty cell, and win by completing a path of their pieces between either their sides of the board (Hex and Quax) or all sides of the board (Y). Each of the games shown in Fig. 1 has been won by White.

An attractive feature of Hex and Y is that there can be no ties; exactly one player must win. This is due to the trivalent nature of the hexagonal tiling, which unlike the square grid has no diagonal neighbors (explained in detail in [1]). Quax, invented by New Zealand mathematician Bill Taylor in 2000, has an additional rule that players may place a bridge between diagonal pieces of their color, to avoid deadlocks on the square grid [2].

Connection games are described as *variable geometry* games, which means that the size and shape of the connection does not matter; it is the mere fact of connection that counts. For this reason, connection games can be played on a wide variety of board designs, and tend to scale well to different board sizes without rule changes.

2. Quadrant games

The idea of recursively-defined connection games has occurred to a number of people, but Steven Meyers, a game designer from Cincinnati, USA, was the first to suggest a workable set of rules with his game Quadrant Hex in 2000 [3].

Fig. 2 shows the previously mentioned game of Hex realized as a game of Quadrant Hex. The main board (called the *supergame*) is split into four quarter-sized boards (called the *subgames*) and a game of Hex is played in each according to the same rules. Players still only place one piece per turn on the supergame; each move is implied on the relevant subgame. In other

E-mail address: Cameron.Browne@swishzone.com.

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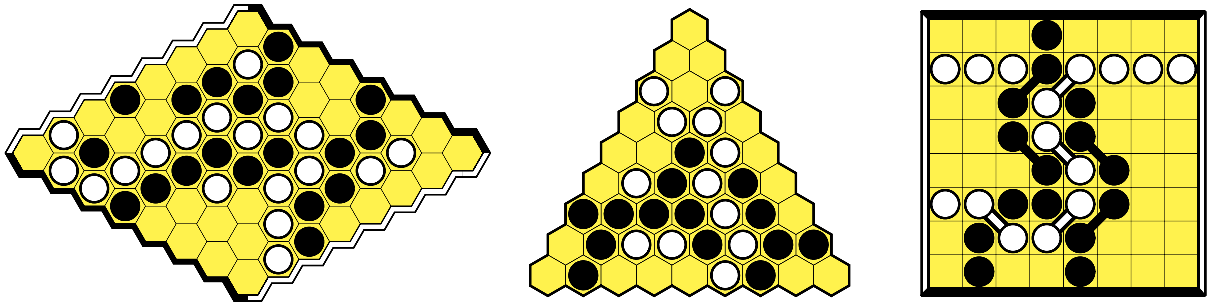

Fig. 1. Games of Hex, Y and Quax, each won by White.

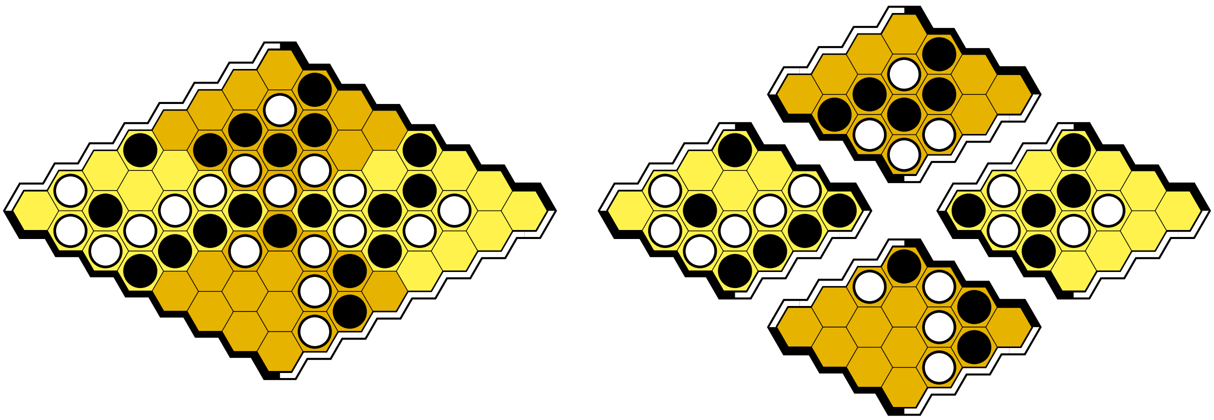

Fig. 2. A game of Quadrant Hex won by Black (3–1).

words, the single piece placed each turn is effectively placed once on every level of subdivision.

Quadrant Hex is best played with an even number of cells along each side of the supergame to avoid overlap between subgames. The colored board regions serve no purpose except to make the subdivision explicit to the players; implicit subdivision on a uniform board is perfectly legitimate but is more difficult for the players.

Quadrant Hex is won by the player who wins the majority of the five component games. Play does not necessarily stop as soon as the supergame is won, and in fact winning the supergame has no more bearing on the outcome than winning a subgame. This is shown in Fig. 2, where Black has won the overall match by winning three component games (all subgames) while White has only won one component game (the supergame). Note that it is not necessary to play all component games to completion to determine the winner. Like Hex, exactly one player must win in Quadrant Hex.

This recursive subdivision principle constitutes a metarule that can be applied to most connection games played on a regularly tiled board whose shape constitutes a *rep-tile* (a polygon that can be dissected into smaller copies of itself [4]). Quadrant Hex should technically be called Quarter Hex, but the term *quadrant* will continue to be used for historical reasons.

Fig. 3 shows the previously mentioned game of Y realized as a game of Quadrant Y, using the same recursive principle. Again, Black wins the overall match by winning three subgames, even though White has won the supergame and one subgame.

Note that the central Quadrant Y subgame is surrounded and shares an edge with each of the three adjacent subgames, so that a piece on the shared edge of one subgame is also implied on the other. This is necessary if all four subgames are to be of equal size, and means that the Quadrant Y supergame *must* have an odd number of cells along each side.

Fig. 4 shows the previously mentioned game of Quax realized as a game of Quadrant Quax. The overall match has been won by White, who has won the supergame and two subgames. Black has only won two subgames.

Note that bridges between pieces in adjacent subgames do not survive the subdivision process. Luckily this does