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Domain construction for volumetric cross-parameterization

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ABSTRACT

We present an algorithm in this paper for constructing volumetric domains with consistent topology to parameterize three-manifold solid models having homeomorphic topology. The volumetric parameterizations generated by our approach share the same set of base domains and are constrained by the corresponding anchor points. Our approach allows users to control interior mappings by specifying interior anchor points, and the anchor points are interpolated exactly. With the help of a novel construction algorithm developed in this work, the volumetric cross-parameterization computed by our method demonstrates its functionality in several examples.

1. Introduction

Many geometric processing applications require a bijective mapping between models (for example, texture mapping, detail transfer, morphing, and shape analysis). Computing a bijective mapping between two-manifold surfaces has been widely studied in computer graphics. A general solution for constructing such mappings can be computed through global parameterization approaches (such as [1–3]). However, for the applications such as morphing and detail transfer, parameterization must be constrained by semantic features, which are usually specified as anchor points on the surfaces of input models. In surface parameterization approaches [4–7], common base-domains are constructed for the surfaces of input models to establish mappings that satisfy the constraints prescribed by anchor points.

The information provided by the boundary surfaces of a model may be insufficient for describing interior information like material, intensity, and micro-structure, which should be defined in the entire solid model. Therefore, researchers have paid more attention to volumetric parameterization recently. Similar to the surface cases, volumetric cross-parameterization is also constrained by semantic features. Computing a constrained mapping between three-manifold models is more challenging than between two-manifold models. Some existing approaches [8–12] formulate the computation of volumetric mapping globally based on surface correspondences. Bijective mappings can be obtained in some specific types of domain shapes (such as star shape by the method of Xia et al. [12]). In real applications, models can have complex geometry, nontrivial topology, and even interior structures. The global domain of such models must be decomposed into sub-domains with simpler shapes to ensure bijection in mappings. This motivates our work to find a solution for constructing a complex of base-domains with consistent topology. Directly computing a volumetric mapping by using the result of surface cross-parameterization as constraints cannot guarantee the result of bijective mapping. For instance, the radial basis functions (RBFs) based mapping presented in [13] can have self-intersection. Similar problem occurs when tetrahedral mesh based deformation [14] is applied to generate the volumetric mapping. Warping a volumetric mesh (with 171.7 k tetrahedra) for the cylinder in Fig. 2 to the rabbit leads to 15 k degenerated tetrahedra.

In this paper, we propose a method to compute domains of volumetric cross-parameterization on three-manifold models having homeomorphic topology, where the parameterization is constrained by anchor points. Here, a few heuristics are applied to specify anchor points for generating successful volumetric mapping:

- First, the anchor points should be defined according to the semantic correspondences – e.g., mapping two toes of a human model to the ears and shoulders of another human body will generate highly distorted mapping.
- Second, relatively uniform distribution of anchor points is expected to result in base-domains with good shape, which is helpful to reduce the stretch in mapping.
- Third, anchors are specified to help decompose the domains into nearly convex regions and help add constraints inside solids.

The computation is conducted on solid models represented by tetrahedral meshes, which can be generated from an intersection-free triangular mesh model (e.g., by the publicly available tool in [15]).

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A tetrahedral mesh M can be represented by a tuple $M=(V,E,F,T)$ which is a collection of vertices (V), edges (E), faces (F), and tetrahedra (T). In addition to the tuple, a solid model M to be parameterized also has a set of pre-defined anchor points G, which could be specified on the boundary surface of M or inside M. The models are then to be parameterized onto a set of *common base-domains* (CBD). The CBD can be considered as a volumetric mesh model formed by polyhedral cells, where every vertex of the cells is corresponding to an anchor point. By this setup, the volumetric cross-parameterization on a set of models M_i $(i=1,2,\ldots)$ sharing the same set of anchor points G is converted into a problem of how to consistently partition a model into a set of *curved polyhedral* regions (called *C-polys* in this paper) according to the CBD. The template of CBD is also represented by a tuple $\Psi=(G_\Psi,E_\Psi,F_\Psi,D_\Psi)-G_\Psi$ is the set of vertices, E_Ψ and F_Ψ are the collections of edges and faces, and D_Ψ is the set of polyhedral cells (see Fig. 1 for the base-domains of a genus-two model).

The basic idea of domain construction on a given model M is to compute curves, patches, and C-polys in M between anchor points according to edges E_Ψ, faces F_Ψ and polyhedral T_Ψ in Ψ respectively. The connectivity of constructed C-polys needs to be consistent with D_Ψ. When all models are parameterized to the same set of CBD, the cross-parameterization between them is established (e.g., our examples use the mean value coordinates [16]). Fig. 2 shows an example of how to establish cross-parameterization in the volumes of models by our domain construction algorithm.

1.1. Related work

Surface parameterization has been studied for many years and comprehensive surveys can be found in [17,18]. Here, our review only focuses on the approaches that find constrained bijective mappings between a pair (or a set) of models. Praun et al. [4] used the connectivity of a predefined template as base-domains and traced the boundary of patch layouts on each of the input meshes by linking the given anchor points in a consistent way as that in the template. Kraevoy and Sheffer [5] and Schreiner et al. [6] further extended the idea of Praun et al. [4] by automating the generation of common base-domains. Our work presented in this paper generalizes the cross-parameterization to volumetric domains.

Volumetric parameterization plays an important role in many solid modeling applications and has attracted more and more attention recently. Ju et al. [19] and Floater et al. [16] extended the mean-value coordinates [20] from surfaces to volumes to compute the interpolation of volumetric data. Mean-value coordinate is a powerful and flexible tool to compute the mapping between two volumes. However, to use it on general solid models with complex shape, a domain construction method as what we propose in this paper is needed.

Wang et al. [10] generalized the harmonic mapping to tetrahedral meshes and reduced the discrete harmonic energy by a variational procedure. They proposed an algorithm to map a genus-zero volume

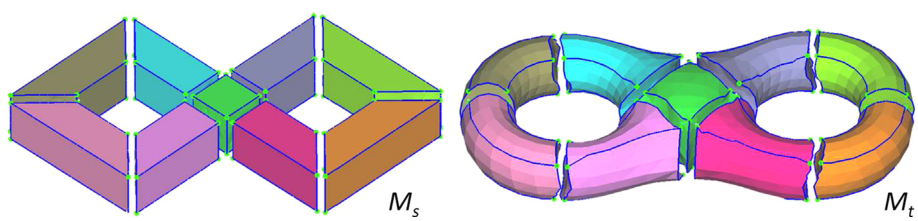

Fig. 1. The polyhedral cells in the template set of base-domains (left) and their corresponding C-polys (right), where the C-polys corresponding to different base-domains are displayed in different colors. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

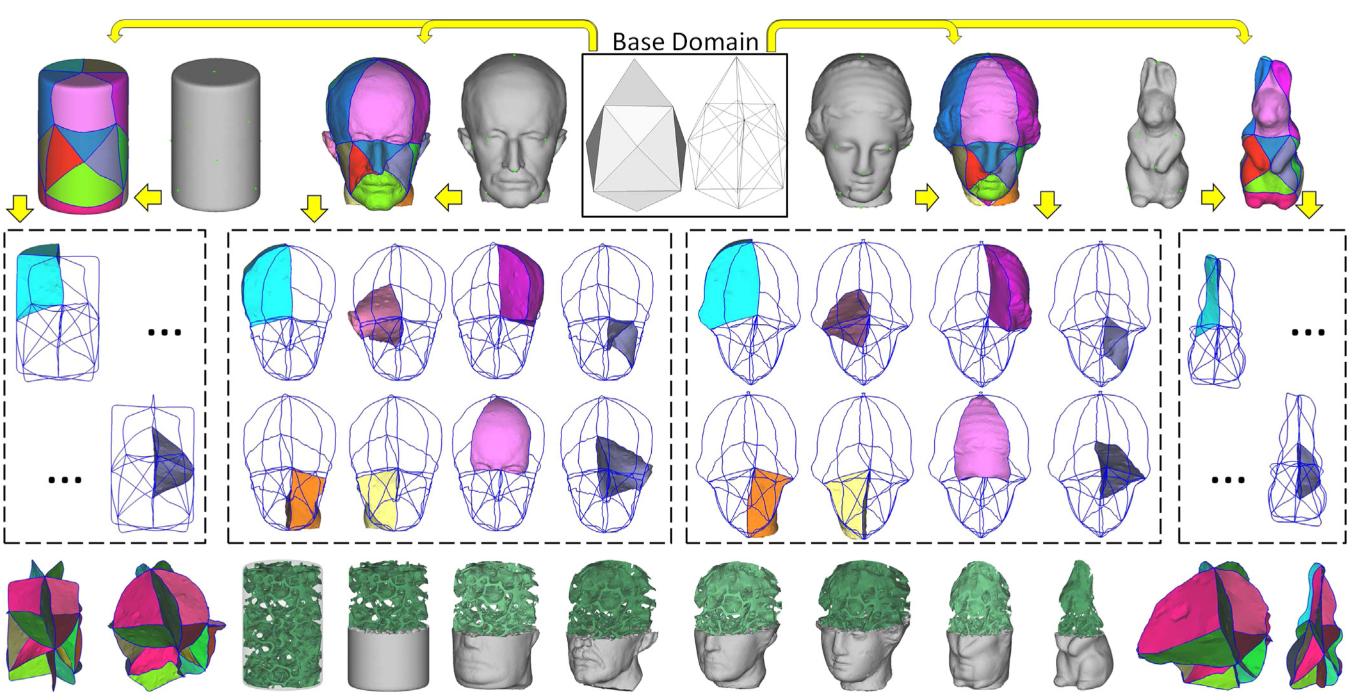

Fig. 2. Volumetric cross-parameterization can be established among models with different shapes. Given the prescribed anchor points, the entire solid model can be decomposed into curved polyhedral cells having the same connectivity as that of the common base-domains. By the volumetric cross-parameterization, interior structures of the cylinder model can be transferred across all the models (bottom row).