Computers & Graphics 38 (2014) 108–116

Contents lists available at ScienceDirect

Computers & Graphics

journal homepage: www.elsevier.com/locate/cag

Special Section on CAD/Graphics 2013

Efficient schemes for joint isotropic and anisotropic total variation minimization for deblurring images corrupted by impulsive noise

Yong Li [a,b], Zhangjin Huang [a,b,*]

[a] School of Computer Science and Technology, University of Science and Technology of China, PR China
[b] Key Laboratory of Software in Computing and Communication, Anhui Province, PR China

ARTICLE INFO

Article history:
Received 5 August 2013
Received in revised form
6 October 2013
Accepted 19 October 2013
Available online 5 November 2013

Keywords:
Total variation
Isotropic
Anisotropic
Image restoration
Augmented Lagrangian method
Fast Fourier transform
Metric Q
Singular value decomposition

ABSTRACT

Total variation (TV) model is a classical image restoration model. The introduction of this model is revolutionary, since TV can preserve discontinuities (edges) while removing other unwanted fine scale details. Lots of efficient methods have been successfully devised and applied to image restoration. However, many of them are sensitive to numerical errors. In this paper, we will first introduce a robust TV based model, which regularizes the restoration using joint isotropic and anisotropic total variation to suppress numerical errors, then present an efficiently iterative algorithm using augmented Lagrangian method. By separating the problem into three sub-problems, the algorithm can be solved efficiently either via fast Fourier transform (FFT) or closed form solution in each iteration. Finally, we use metric Q which is based upon singular value decomposition of local image gradient matrix to effectively measure true image content. Extensive numerical experiments demonstrate that our proposed model has better performance than several state-of-the-art algorithms in terms of signal–noise ratio and recovered image quality.

1. Introduction

Images are produced to record or display useful information. Due to the imperfections in the imaging and capturing process, however, the final image invariably represents a degraded version of the original scene. The undoing of these imperfections, i.e., image restoration, is critical to many of the subsequent image processing tasks. Image restoration (sometimes referred to as image deblurring or image deconvolution) is concerned with the reconstruction or estimation of the uncorrupted image from a blurry and noisy one.

Total variation (TV) regularization has been successfully applied to image restoration and extensively generalized [1–12], since it was first introduced by Rudin, Osher and Fatemi (ROF) in their pioneering work [13] on edge preserving image denoising. It was designed with the explicit goal of preserving sharp discontinuities in images while removing noise and other unwanted fine scale details. Variational models formulate the solution of image restoration problem as minimizers of appropriately chosen functionals [34]. The minimization technique for such models routinely involves the solution of nonlinear partial differential equations (PDEs) derived as necessary optimality conditions. Hence, this kind of model is difficult to solve due to the TV term's nonlinearity and non-differentiability. Many fast solvers [1,14–21] have been designed for TV minimization with squared L^2 fidelity term (TV-L^2 model), which is particularly suitable for deblurring images corrupted by Gaussian noise.

Besides Gaussian noise, impulsive noise is another typical and important noise. Impulsive noise is often generated by malfunctioning pixels in camera sensors, faulty memory locations in hardware, or erroneous transmission [22]. Impulsive noise has two common types: salt-and-pepper and random-valued noise. In images contaminated by such noise, a certain number of pixels of the underlying image are uncorrupted, and the corrupted pixels usually have intensities distinguishable from those of their neighbors. TV-L^1 model, which uses TV with L^1 fidelity term, was devised to restore images corrupted by impulsive noise. Compared with TV-L^2 model, TV-L^1 has many advantages. First, it is more suitable for impulsive noise removal [23,24]. Second, it fits uncorrupted pixels exactly and regularizes corrupted pixels perfectly. Finally, it provides many useful properties [25–27]. However, the TV-L^1 model is hard to compute due to the nonlinearity and non-differentiability of both the TV term and the data fidelity term. Some existing numerical methods include gradient descent method [28], Lagrangian-based alternating direction method [31], the splitting-and-penalty based method [33], and the primal–dual method [29] based on semi-smooth Newton algorithm [32], as well as alternating direction method [30].

FTVd [20,33] and ALM [1] are two of the state-of-the-art algorithms for TV-L^1 image reconstruction. FTVd applied the

* Corresponding author.
E-mail addresses: liyong89@mail.ustc.edu.cn (Y. Li), zhuang@ustc.edu.cn (Z. Huang).

0097-8493/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.cag.2013.10.019

well-known variable-splitting and quadratic penalty function techniques in optimization. The per-iteration computational complexity of the algorithm is three fast Fourier transforms [20,33]. The latest FTVd version 4.1 uses alternating direction method (ADM) to avoid the ill-conditioning caused by large penalty parameter in previous versions and achieve faster convergence. FTVd is applicable to recover blurry images corrupted by Gaussian noise (TV-L^2 model) or impulsive noise (TV-L^1 model). ALM extended augmented Lagrangian method for TV-L^2 restoration to TV models with non-quadratic fidelities [1]. ALM applies to reconstruct images with impulsive noise (TV-L^1 model) or Poisson noise (TV-KL model). For these two cases, ALM is extremely efficient since all the sub-problems have closed form solutions.

However, these two algorithms both solve isotropic (2-norm based) TV-L^1 models. When corrupted with high level impulsive noise, images recovered with ALM or FTVd tend to be too smooth and lose small scale features. What is worse, restored images may appear some random dirty points (see Fig. 7).

In this paper, we propose a new model which uses joint isotropic-and-anisotropic total variation as regularizer. Augmented Lagrangian method for isotropic TV-L^1 model is first extended to anisotropic (1-norm based) TV-L^1 model. Then an approximate algorithm to solve the mixed TV-L^1 model is described. Numerical results on images with different levels of impulsive noise demonstrate that the new algorithm can suppress dirty points and produce higher quality restorations than FTVd and ALM.

The rest of this paper is organized as follows. In Section 2, we propose a novel mixed TV-L^1 model, devise augmented Lagrangian method for anisotropic TV-L^1 model, and describe an efficient algorithm to solve the proposed model. Numerical results in comparison with FTVd and ALM are presented in Section 3. Finally, concluding remarks are given in Section 4.

2. Mixed TV-L^1 image restoration

We consider the problem of recovering grayscale images degraded by blurring and impulsive noise (e.g., salt-and-pepper noise). Without loss of generality, we assume that the underlying images have square domains. Let $\overline{u} \in R^{N^2}$ be an original $N \times N$ grayscale image, $K \in R^{N^2 \times N^2}$ represent a blurring operator, $n \in R^{N^2}$ be an additive noise, and $f \in R^{N^2}$ be an observation which satisfies the relationship:

$$f = K * \overline{u} + n.$$

Given f and K, the image $\overline{u}$ is restored from the following model:

$$\min_u R(\nabla u) + \alpha \| Ku - f \|_1, \tag{1}$$

where $R(\nabla u)$ is a regularization term, ∇u denotes the discrete gradient of u at pixels, and $\alpha > 0$ balances the regularization term and the L^1 fidelity term.

For isotropic TV-L^1 model, the regularization term is

$$R(\nabla u) = TV_I(u) = \sum_{1 \le i,j \le N} \| (\nabla u)_{i,j} \|_2.$$

For anisotropic TV-L^1 model, the regularization term is

$$R(\nabla u) = TV_A(u) = \sum_{1 \le i,j \le N} \| (\nabla u)_{i,j} \|_1.$$

Through extensive numerical experiments, we found that ALM [1] and FTVd [20] failed to reconstruct images seriously corrupted by impulsive noise. The recovered images may contain some random dirty points. We conjecture that isotropic total variation term is sensitive to numerical errors and the stopping criterion (mean-squared error, i.e., MSE) is not sufficient to measure true image content.

Therefore, we combine isotropic and anisotropic TV regularization terms to get a mixed TV regularizer:

$$R(\nabla u) = TV_I(u) + TV_A(u) = \sum_{1 \le i,j \le N} \| (\nabla u)_{i,j} \|_2 + \sum_{1 \le i,j \le N} \| (\nabla u)_{i,j} \|_1.$$

And we utilize metric Q [35] instead of MSE to measure the true image content and decide when to stop the algorithm.

In the following, the isotropic TV-L^1, anisotropic TV-L^1 and joint TV-L^1 models are referred to as TV_I-L^1, TV_A-L^1, and TV_J-L^1 models, respectively.

2.1. Metric Q

Usually, MSE is used as the metric for measuring the closeness of two variables and determines when to stop the algorithm. However, MSE does not take the structure of an image into account and stops the algorithm at the wrong time. Therefore, we turn to metric Q [35], and this measure is properly correlated with noise level, sharpness and intensity contrast of the structured regions of an image.

As we all know, image structure can be measured effectively by image gradients. For an image u, the gradient matrix over an $N \times N$ window W is denoted as G. The corresponding gradient covariance matrix is $C = G^T G$. By calculating the local dominant orientation via computing singular value decomposition of G, we can get important information about the content of the image patch W [36,37]

$$G = USV^T = U \begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix} [v_1 \ v_2]^T. \tag{2}$$

where U and V are both orthonormal matrices. Vector v_1 represents the dominant orientation of the local gradient field, and v_2 describes the dominant "edge orientation" of this patch. The singular values $s_1 \ge s_2 \ge 0$ represent the energy in the directions v_1 and v_2. Then metric Q [35] is defined as

$$Q = s_1 \frac{s_1 - s_2}{s_1 + s_2}. \tag{3}$$

Fig. 1 shows that higher SNRs can be obtained using metric Q other than MSE in image restoration.

2.2. Augmented Lagrangian method for TV_A-L^1 restoration

We extend augmented Lagrangian method for TV_I-L^1 restoration [1] to TV_A-L^1 model. Two auxiliary variables $p \in S$, $v \in V$ are introduced to eliminate the nonlinearity for u, where V represents

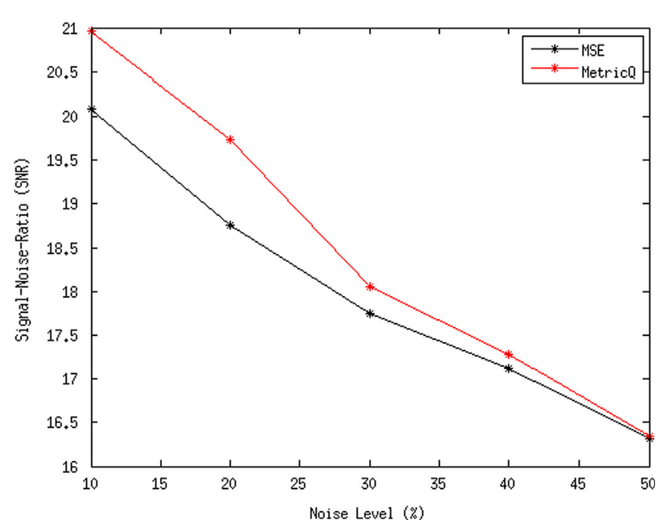

Fig. 1. Signal–noise-ratio of lena obtained by using different stopping rule: MSE and metric Q.