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Footpoint distance as a measure of distance computation between curves and surfaces

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ABSTRACT

In automotive domain, CAD models and its assemblies are validated for conformance to certain design requirements. Most of these design requirements can be modeled as geometric queries, such as distance to edge, planarity, gap, interference and parallelism. Traditionally these queries are made in discrete domain, such as a faceted model, inducing approximation. Thus, there is a need for modeling and solving these queries in the continuous domain without discretizing the original geometry. In particular, this work presents an approach for distance queries of curves and surfaces, typically represented using NURBS.

Typical distance problems that have been solved for curves/surfaces are the minimum distance and the Hausdorff distance. However, the focus in the current work is on computing corresponding portions (patches) between surfaces (or between a curve and a set of surfaces) that satisfy a distance query. Initially, it was shown that the footpoint of the bisector function between two curves can be used as a distance measure between them, establishing points of correspondence. Curve portions that are in correspondence are identified using the antipodal points. It is also identified that the minimum distance in a corresponding pair is bound by the respective antipodal points. Using the established footpoint distance function, the distance between two surfaces was approached. For a query distance, sets of points satisfying the distance measure are identified. The boundary of the surface patch that satisfies the distance is computed using the α-shape in the parametric space of the surface. Islands contributing to the distance query are also then computed. A similar approach is then employed for the distance between a curve and a set of surfaces. Initially, the minimum footpoint distance function for a curve to a surface is computed and repeated for all other surfaces. A lower envelope then gives the portions of the curves where the distance is more than the query.

1. Introduction

Freeform objects such as curves/surface are predominantly used in automotive domain. The CAD models of body parts are validated for conformance to certain design requirements. Geometric queries, such as distance to edge, planarity, gap, interference and parallelism, are typically used for modeling design requirements. These properties are usually computed on the approximated models, using discrete approaches such as faceting (or meshes) rather than on the freeform representation itself. There are certain disadvantages associated with this approach, such as

- Approximation: Since faceted models are only approximate representation of original geometry, the queries made on them are also approximate.
- Computational complexity: Accuracy of results depends on the quality of faceting. There is a tradeoff as computational expense increases with densely faceted models.
- Result remapping: Mapping of results from faceted representation to original geometry adds to the approximation.

Though commercial CAD packages offer some capability to make geometric queries from original geometry (exact computation), they are elementary in nature and difficult to scale. For example, given two geometric entities one can query corresponding points where the distance is minimum, but one cannot query for corresponding regions where the distance is less than a threshold value. For spot welding operation one may identify regions where faces of welded parts are within a given proximity of each other. Such regions can be

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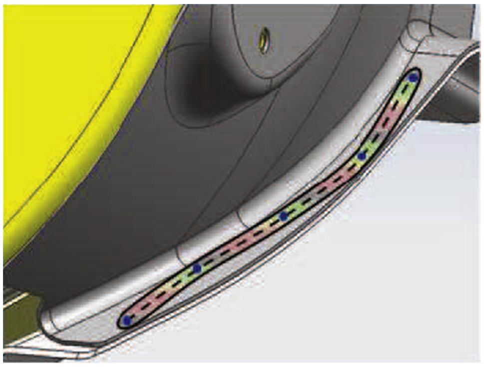

Fig. 1. Manufacturing workspace for creating valid joint entities.

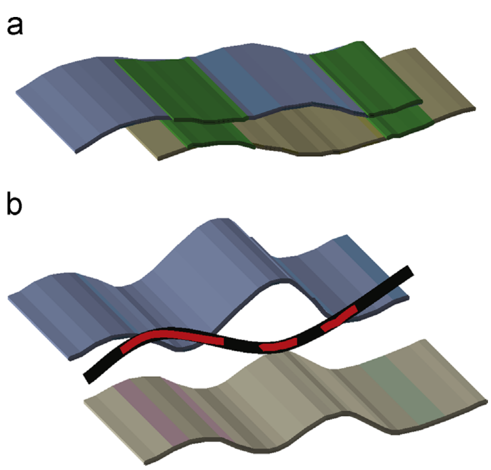

Fig. 2. Distance for surface–surface or curve and a set of surfaces. (a) Two surfaces, where the corresponding patches (in green) satisfy a distance threshold. (b) A curve and a set of surfaces, where the red segments of the curve satisfy a distance threshold with respect to all the surfaces. (For interpretation of the references to colour in this figure caption, the reader is referred to the web version of this paper.)

defined as manufacturing workspace within which valid joint entities, such as weld spots, laser weld curves and adhesive bonding strips, can be created (Fig. 1). Thus, there is a need to develop an efficient method for modeling and solving these queries accurately. Rather than making the queries through discretized models, in the current work, the objective is to model distance queries without approximating the inputs by a faceted model.

1.1. Problem statement

In this work, the following are the problems that are looked at:

- Given two freeform surfaces, compute regions on each surface, such that, for any point (P) in a region on one surface there lies a corresponding point (P') on the other surface at a distance less than a threshold value (Fig. 2(a)).
- Given a freeform curve and a set of freeform surfaces, compute segments of the curve where the minimum distance between the curve and any of the surfaces is more than a threshold value (Fig. 2(b)).

1.2. Related work

Precise computation of freeform curves and surfaces without discretizing them is now becoming prominent, as the accuracy of such computations is provenly better. One of the prominent approaches is to formulate the problem in terms of parametric representation and then solved in parametric space [1], employing a rational solver [2]. This approach has been demonstrated for various algorithms such as bisectors [3], Voronoi cell [4], medial axis [5], and minimum enclosing sphere [6]. Another approach for global analysis of freeform geometry is a dimensional lifting scheme, where the problem is solved in higher dimensions and has been demonstrated for visibility problems [7], minimum distance query [8], and critical point analysis [9].

In the precise computation of distance function of curves and surfaces, most of the work has focussed on computing the minimum distance between a point and a curve/surface. For a pair of curves, only [10] deals with distance between two curves, albeit Hausdorff distance, which does not appear to be useful for the problems at our hand.

1.3. Contributions

To the best of our knowledge, no work seems to exist that computes corresponding patches of curves/surfaces satisfying above or below a certain distance value, which is the focus of this work. Perhaps this is the first time that such a distance query has been addressed. Moreover, the curves and surfaces are not discretized into mesh. The following are the major contributions:

- Footpoint distance measure has been proposed as a measure for distance computation.
- Points of correspondence through footpoints were explored in the case of curve–curve case and found to be a useful tool.
- Corresponding surface patches for the surface–surface case are identified using footpoint distance. α-Shape has been used to detect boundaries including island regions.
- A lower envelope based approach has been proposed and demonstrated for the distance query between a curve and a set of surfaces.

2. Establishing footpoint distance as a measure

To solve the stated problems in Section 1.1, we started solving 'corresponding curve portions between two given curves'. In this section, computing distance between two curves is explained, along with how to employ them to get the distance bounds for the curve portions. This then served as motivation to solve the problems delineated in Section 1.1.

Definition 1. An *Entity* is a freeform curve or surface.

Definition 2. Bisector point is a point that is equidistant from two different points on the two entities. A set of bisector points is then called as bisector (which could be a curve or a surface).

Definition 3. Footpoint is a point on an entity corresponding to its bisector point. Footpoint distance (FD) is the Euclidean distance between the footpoints corresponding to a bisector point.

For example, in Fig. 3, footpoints f_1 and f_2 correspond to a bisector point m. The circle with centre m and radius $\|m-f_1\|$ is a

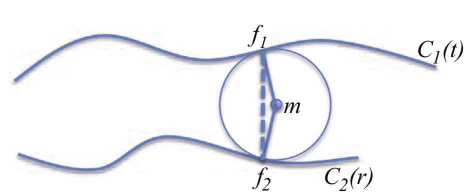

Fig. 3. Illustration of footpoint distance function. f_1 and f_2 are the footpoints corresponding to the bisector point m. Distance between the footpoints is called the footpoint distance.