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General image denoising framework based on compressive sensing theory

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ABSTRACT

Image denoising is an important issue in many real applications. Image denoising can be considered to be recovering a signal from inaccurately and/or partially measured samples, which is exactly what compressive sensing accomplishes. With this observation, we propose a general image denoising framework that is based on compressive sensing theory in this paper. Most wavelet-based and total variation based image denoising algorithms can be considered to be special cases of our framework. From the perspective of compressive sensing theory, these algorithms can be improved. To demonstrate such an improvement, we devise four novel algorithms that are specialized from our framework. The first algorithm, which is for the synthetic case, demonstrates the considerable potential of our framework. The second algorithm, which is an extension of wavelet thresholding and total variation regularization, has better performance on natural image denoising than these algorithms. The third algorithm is a more sophisticated algorithm for natural image with Gaussian white noise. The last algorithm addresses Poisson-corrupted images. Compared with several state-of-the-art algorithms, our intensive experiments show that our method has a good performance in PSNR (peak signal-to-noise ratio), fewer artifacts and high quality with respect to visual checking.

1. Introduction

Image denoising is a classical image processing problem. In recent years, many advanced methods for image denoising have been proposed, including total variation image regularization [1,2], PDE-based image diffusion [3,4], wavelet thresholding [5], bilateral filtering [6,7], non-local means [8,9], basis pursuit denoising [10], BM3D [11], among others. These methods can perform image smoothing/denoising as well as preserve edges to a certain extent. However, each of these methods has certain shortcomings in terms of image quality or computational efficiency. Non-local means usually produce good image quality but at a relatively high computational cost. PDE-based image diffusion and total variation methods tend to result in piecewise constant regions on the images, although they preserve strong edges. Wavelet thresholding and basis pursuit denoising methods can reduce noise because most natural images have sparse representation when expressed in wavelets or a set of bases. However, they are likely to create ripple artifacts. Bilateral filtering is an effective image denoising method that is easy to implement, but it has not yet attained a desirable level of applicability in terms of image quality. For details of the techniques and performance comparisons, readers can refer to the survey paper in [12].

In this paper, we propose a general image denoising framework motivated by the total variation method, wavelet thresholding and compressive sensing theory [13,14]. The word "general" here highlights the fact that many existing methods can be viewed as special cases of our denoising framework. Our framework is sufficiently flexible for addressing a wide range of images and noise. With a carefully designed configuration, our specialized algorithms can attain better performance in terms of the image quality or computational complexity compared with several state-of-the-art image denoising algorithms. In Sections 5–7, we will present the four specialized algorithms to demonstrate the potential, flexibility and robustness of our framework.

The contributions of this paper are summarized as follows:

1. A general image denoising framework is proposed. Many previous algorithms can be considered to be special cases of our framework. In other words, we can generalize many existing approaches.
2. Under this framework, we can overcome certain shortages that are present in existing approaches and obtain an improvement in PSNR (peak signal-to-noise ratio) or visual quality.
3. Our framework is based on Compressive Sensing theory, which enables us to introduce new insights into existing image denoising algorithms.

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4. From our framework, we derive four novel, specialized algorithms. One algorithm is for the synthetic case, two of the remaining algorithms are for natural images with Gaussian white noise, and the last algorithm addresses Poisson-corrupted images.

2. Related work

Image denoising is a classical image processing problem that has been studied for several decades. There has been a large volume of literature dedicated to solving this problem. However, providing an in-depth survey on this topic is not the primary focus of this work. In short, we can briefly categorize image denoising techniques into two classes that are closely related to our framework: wavelet-based and regularization-based algorithms.

In the wavelet domain, the energy of a natural signal is concentrated in a small number of coefficients; noise is, however, spread over the entire domain. Hence, one common approach is to reduce noise by thresholding the coefficients in the wavelet domain [5]. For wavelet thresholding, a variety of techniques have been proposed, for example, hard or soft, multiscale thresholding [15], translation-invariant thresholding [16], and threshold-selection methods, such as SURE thresholds [17]. Thresholding based on orthogonal or biorthogonal wavelet transforms tends to create ripple artifacts, while translation-invariant thresholding approach can usually help to reduce these artifacts.

Image regularization is usually used to restore the latent image from blurriness or degradation. If we assign the blur kernel to be an identity matrix, then the image restoration process becomes an image denoising problem. Typically, image regularization is usually composed of two terms: the fitness term and the regularization term, which are as follows:

$$\arg\min_u \frac{1}{2}\|u-f\|^2+\lambda J(u) \tag{1}$$

where f is the noisy image, and $\|\cdot\|$ denotes the l^2 norm. The role of the regularization term $J(u)$ is to encourage or constrain the solution of (1) to have a certain property. For example, if $J(u)$ is the total variation of u, then (1) is the famous total variation regularization, which decreases the total variation of the image to remove the noise. In addition, we can have many choices for $J(u)$ [18], for example, we can use the l_2 norm (i.e., Tikhonov regularization), maximum entropy regularization and l_1 norm.

Recently, researchers have focused increasing attention on l^1 regularization because it is closely related to wavelet thresholding and compressive sensing. In fact, if $J(u)$ is $\|\Psi_u\|_1$, where Ψ is a tight frame, then (1) is equivalent to wavelet soft thresholding. From the perspective of compressive sensing theory, $\|\Psi_u\|_1$ is an approximation of $\|\Psi_u\|_0$, which makes Ψu sparse. We usually want Ψu to be sparse, but this characteristic cannot be obtained by l^2 regularization. Hence, l^1 regularization is also called sparse analysis approximation. Correspondingly, there is another approximation approach called *sparse synthesis approximation* (also called the basis pursuit denoising (BPND) method [10]) which assumes that the signal has a sparse synthesis in a dictionary $\mathcal{D}=\{\phi_p\}_{p\in\Gamma}$. If $\mathcal{D}$ is an orthonormal basis, then it is equivalent to a sparse analysis approximation. This situation is, however, not the general case when the dictionary is redundant [19]. In the algorithm, a large $\mathcal{D}$ is likely to result in over-fitting, which means that part of the noise is fitted by certain functions in the dictionary $\mathcal{D}$. As a result, this part of noise is included in the denoised result, and ripple artifacts arise.

When using wavelet hard thresholding, we can usually obtain a higher PSNR than with soft thresholding, but it is more likely that ripple artifacts are created. In this regard, image regularization is dedicated to avoiding oscillatory and ripple artifacts. The specialized algorithms we proposed in our framework, see Section 6, combine the unique strengths of these two approaches such that we can obtain higher flexibility and robustness while attaining a higher image quality.

3. Compressive sensing theory

The theory of compressive sensing (CS) demonstrates how a subsampled signal can be faithfully reconstructed through optimization techniques. If a signal x is sparse in a basis (or frame) Ψ, then it can be perfectly reconstructed from fewer measured samples y with a very high probability, through the following optimization [20]:

$$\arg\min_x \|\Psi x\|_1 \quad \text{subject to } \Phi x = y \tag{2}$$

where Φ is the measurement matrix.

In many practical applications, we cannot assume that the signals are strictly sparse but still compressible. More precisely, we assume that the nth largest entry of its coefficients in a frame Ψ obeys $|\Psi(x)|_{(n)} \le R\cdot n^{-1/p}$, where $R>0$ and $p>0$. In this case, there is a similar result that states that the signal x can be approximately reconstructed with a high probability through the following non-linear optimization [21]:

$$\arg\min_x \frac{1}{2}\|\Phi x-y\|^2+\lambda\|\Psi x\|_1 \tag{3}$$

where $\lambda>0$ is the regularization parameter that trades off between the fitness term and the regularization term. There are many numerical algorithms for the optimization problems (2) and (3), such as NESTA [22], TwIST [23], FPC [24], and IHT [25], to name a few.

It should be noted that not all Φ and Ψ are suitable for compressive sensing. CS theory prefers a pair that has a low coherence [13] which is defined by

$$\mu(\Phi,\Psi)=\sqrt{N}\cdot \max_{1\le j\le N}|\langle\varphi_k,\psi_j\rangle| \tag{4}$$

A coherence ranges from 1 to $\sqrt{N}$ and reaches its maximum when Φ and Ψ are the same.

4. General image denoising framework

It is well known that many natural images have a compact representation when expressed in a convenient basis, such as with wavelets. If we can obtain more precise measurements of the original images than the corresponding noisy images, then we can reconstruct the image well by CS theory. Thus, we propose a general image denoising framework as follows:

$$\arg\min_u \frac{1}{2}\|\Phi u-\Phi f\|^2+\lambda\|\Psi u\|_1 \tag{5}$$

where f is the noisy image, and Φ is the measurement operator that separates the noise and the latent image. Note that this framework is very general, thus, we can choose suitable Φ and Ψ to address specific images and noising. In particular, several existing methods can be regarded as special cases of the proposed framework. We divide them into two classes:

Total variation-based algorithms: If Ψ is the gradient operator and Φ is the identical matrix, then Eq. (5) is the total variation (TV) regularization [1]. If Ψ is the gradient operator and Φ is a wavelet hard thresholding operator, then Eq. (5) is the method that is proposed by Durand and Froment [26]. The TV regularization and its variants recover an image that has a gradient vector that is as sparse as possible. From the perspective of CS theory, if the image