Computers & Graphics 38 (2014) 392–398

Contents lists available at ScienceDirect

Computers & Graphics

journal homepage: www.elsevier.com/locate/cag

Special Section on CAD/Graphics 2013

Measuring length and girth of a tubular shape by quasi-helixes

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ARTICLE INFO

Article history:
Received 3 August 2013
Received in revised form
28 October 2013
Accepted 28 October 2013
Available online 15 November 2013

Keywords:
Quasi-helix
Tube shaped objects
Geodesic

ABSTRACT

Length and girth are central to measure the size of tube shaped objects. This paper extends circular helical curves to general tubular shapes and proposes a novel method for measuring their length and girth. We call the extended circular helixes *quasi-helical* curves. A formal definition, as well as a set of practical algorithms for quasi-helixes, is presented in this paper. Experimental results demonstrate that our method is fast, intrinsic, insensitive to noises, invariant to triangulation and resolution. Furthermore, quasi-helical curves can also be used in classifying 3D shapes and designing vector fields on surfaces of revolution.

1. Introduction

Measuring length and girth of tube shaped objects is useful in many research fields including life science, computer vision and industrial engineering. In Applied Physiology, researchers observed exercise effects by automatically tracking of medial gastrocnemius fascicle length [1]. In computer-aided industrial design, girth measurement was used to determine fitting accuracy on digital foot models [2,3]. It is also interesting to estimate weights of digitalized animals based on length and girth measurements [4].

Intuitively, we can simulate the measurement procedure with a tape – finding a point p on a given tube shaped object and computing a shortest closed curve Γ starting from and arriving at p. In fact, Γ is locally shortest everywhere except at p and it gives the girth of Γ's neighborhood. Sometimes people may wind the tape around the object for many times (see Fig. 1) to get the overall girth, i.e., the total length of Γ divided by the winding number. Therefore, it is reasonable to assume that Γ should be a geodesic except at only a few points.

To the best of our knowledge, Chen et al. [5] were the first to propose a posture invariant measurement of human body based on geodesic distance field. Their method takes 5 source points as the input, i.e., the extreme points on the head, hands, and feet of a 3D human model. The skeleton curve is extracted by connecting the barycenters of iso-contours. Finally, the length and girth of each part can be obtained from the skeleton curve and the iso-contours. However, we must point out that iso-contours are not locally shortest and thus significantly different from geodesics; We refer to [6] where Liu et al. systematically discussed the properties of geodesic iso-contours.

In this paper, we propose a novel estimation approach based on the practical measurement procedure. We extend circular helical curves to general tube shaped surfaces. First, we cut the key tubular part into a quad surface $\mathcal{Q}$ with geodesic boundaries, like that achieved in the stripification technique proposed by Liu et al. [7]. After gluing N copies of $\mathcal{Q}$ into a larger geodesic quad $\overline{\mathcal{Q}}$, a geodesic Γ winding around the original tubular surface N times is actually an open shortest path lying on $\overline{\mathcal{Q}}$. In fact, such a curve has a close resemblance to a helix as far as geometric properties are concerned and thus called a *quasi-helical* curve. Γ and its skeleton curve respectively define the girth and length. Quasi-helixes are geometrically invariant to triangulation, resolution and posture change of the input model. Furthermore, quasi-helixes have more applications including vector field design.

This paper is set out as follows. Section 2 reviews related work. We detail the theoretical background of quasi-helixes in Section 3 and the algorithm pipeline in Section 4. Section 5 utilizes experimental results to demonstrate the uses of quasi-helixes. Finally, we discuss the limitations in Section 6 and draw the conclusion in Section 7.

2. Related work

2.1. Previous research in applications of length and girth

In life science field, researchers use the length–girth relationship to study the growth status and distinguish different types of creatures [8,9]. For example, Irvine et al. [10] investigated the influence of length and girth on dive duration of underyearling southern elephant seals. In medical field, experts exploit length,

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http://dx.doi.org/10.1016/j.cag.2013.10.037

girth and width to report the health status of infants [11,12] or give a guidance for physical exercise. Another interesting topic is to predict weight or sex from the length–girth relationship [4]. Length and girth are also useful in computer engineering and computer vision fields.

2.2. Definition of length and girth

To our knowledge, an exact definition of length and girth is unavailable on general 3D surfaces. Zhao et al. [2] suggested intersecting a prescribed girth plane with the given foot model. Their method depends on the embedding space and cannot deal with deforming objects. Some feature aligned parametrization methods [13–15] can also be used to compute length and girth of tube shaped objects, but they are sensitive to noises and local geometric features; see Section 5. Chen et al. [5] proposed a posture-invariant method for automatically computing length and girth based on the iso-contours of the multi-source geodesic distance field. However, the iso-contours are not locally shortest and thus different from practical measurement procedure. Observing that a winding tape is geometrically similar to a circular helix, we present a new type of curve, called *quasi-helix*, to measure length and girth. Compared with existing geometric tools [2,5,15,16], the quasi-helical curves are not only insensitive to noises and local geometric features, but also invariant to triangulation, resolution and posture change.

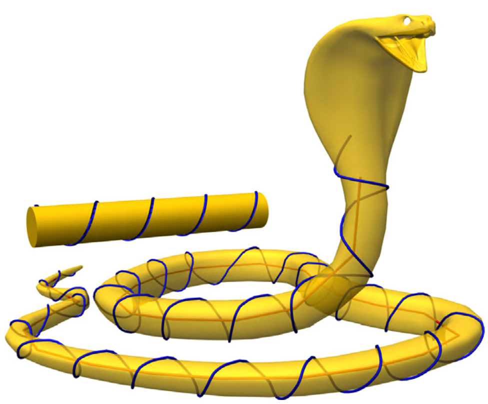

Fig. 1. A circular helix and a quasi-helical curve. The red skeleton curve can be obtained from the quasi-helical curve; see Section 4 for details. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

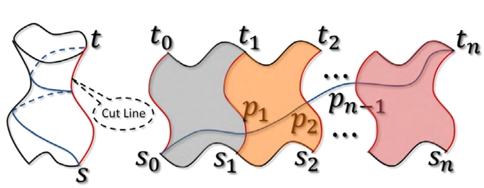

Fig. 2. Proposition 3.

2.3. Computing open and closed geodesics

Sharir and Schorr [17] pioneered an algorithm to compute the "single-source-all-destination" discrete geodesic on convex polyhedra with an $O(n^3 \log n)$ time complexity. Later, Mitchell et al. [18] (MMP) improved the time complexity bound to $O(n^2 \log n)$ by using the "continuous Dijkstra" technique. Chen and Han [19] (CH) suggested building a binary tree to encode all the edge sequences that can possibly contain a shortest path, thereby improving the time complexity to $O(n^2)$. Surazhsky et al. [20] extended the MMP algorithm to compute approximate geodesics with bounded error. Xin and Wang [21] improved the CH algorithm by exploiting a filtering theorem. Liu [22] suggested a speed-up technique for the MMP algorithm [18] based on an observation that the windows on a pair of half-edges can be merged into one structure. Besides the exact algorithms, there are also many algorithms [23–26] to approximate discrete geodesics. Among them, the fast marching method [23], with an $O(n \log n)$ time complexity, has been widely used in the research community.

There are also a few algorithms discussing the closed geodesic problem. Wu and Tai [27] proposed the discretized geodesic curvature flow (dGCF) to compute geodesic loops on triangle meshes using a level set formulation, and later, Zhang et al. [28] improved dGCF by reducing the problem dimension. Xin et al. [29] discussed how to efficiently compute an exact closed geodesic. They also studied a special kind of closed geodesics that are relaxed at some vertex.

3. Theoretical background

Given a smooth surface $\mathcal{S}$, equipped with a Riemannian metric g, and two points $s, t \in \mathcal{S}$, the *shortest* path between s and t is defined to the one with the minimum length among those connecting s and t.

A path $\Gamma(t) \in \mathcal{S}, t \in [0,1]$, is a *geodesic* if and only if $\forall t^* \in [0,1]$, there exists an ε, such that the sub-segment $\underline{\Gamma} = \{\Gamma(t),\ t \in [0,1] \cap [t^* - \varepsilon, t^* + \varepsilon]\}$, is a shortest path on $\mathcal{S}$.

A geodesic may self-intersect, but a shortest path cannot. A shortest path must be a geodesic, but not vice versa. A common technique for computing a shortest path is to (implicitly) find the optimal one among candidate geodesics. Sometimes we may abuse "shortest" and "geodesic" if there is no ambiguity in the context.

Definition 1. A patch $\mathcal{Q}$ is a part of the surface $\mathcal{S}$. $\mathcal{Q}$ is called a *geodesic quad* if $\mathcal{Q}$ is bounded by four shortest paths on $\mathcal{S}$.

Since geodesics are well known to the research community, we directly give the following conclusion without proofs.

Proposition 2. *Given a geodesic quad $\mathcal{Q}$ that is part of the surface $\mathcal{S}$, any shortest path lying in $\mathcal{Q}$ is a geodesic in $\mathcal{S}$.*

As Fig. 2 shows, we cut a tubular surface $\mathcal{S}$, bounded by two shortest loops, into a geodesic quad $\mathcal{Q}$ along the shortest path between s and t. After that, we glue N copies of $\mathcal{Q}$ together and obtain a composite geodesic quad, which is quite similar to the covering technique proposed by Campen et al. [14]. During this

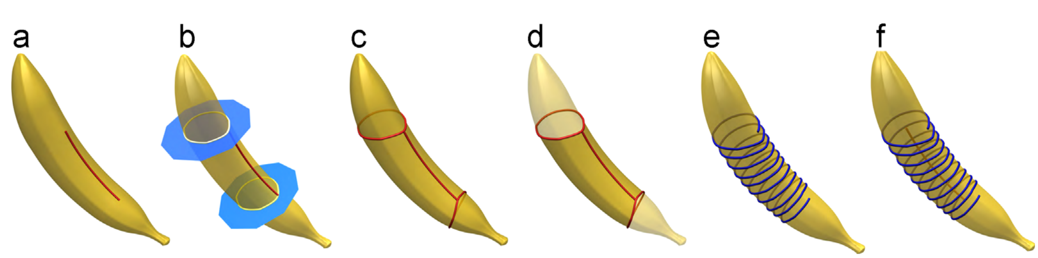

Fig. 3. Algorithm pipeline. (a) Computing the shortest path between two given points, typically feature points. (b) Extract the tubular shape between two section planes. (c) Computing two relaxed shortest paths at the endpoints. (d) Separating the geodesic quad from the remaining part that is actually a topological disk. (e) Computing the quasi-helical curve lying inside the geodesic quad. (f) Extracting the skeleton curve of the tube shaped object.