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Technical Section

Sparsity-based optimization of two lifting-based wavelet transforms for semi-regular mesh compression

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ABSTRACT

This paper describes how to optimize two popular wavelet transforms for semi-regular meshes, using a lifting scheme. The objective is to adapt multiresolution analysis to the input mesh to improve its subsequent coding. Considering either the Butterfly- or the Loop-based lifting schemes, our algorithm finds at each resolution level an optimal prediction operator P such that it minimizes the L_1-norm of the wavelet coefficients. The update operator U is then recomputed in order to take into account the modifications to P. Experimental results show that our algorithm improves on state-of-the-art wavelet coders.

1. Introduction

Wavelets have their roots in approximation theory [1] and signal processing [2] in the late eighties. Since then, wavelets are the most popular technique for representing data in a multiresolution way. They have been used for a vast number of applications: physic, biomedical signal analysis, image processing, and so on. But wavelets have been particularly designed for data coding, because they guarantees compact representation of transformed data, and consequently high compression performances.

In computer graphics, the compact representation is not the sole attractive feature of wavelets. Indeed, current high-resolution acquisition techniques produce highly detailed and densely sampled surface meshes. Not only these massive monoresolution data are difficult to handle and store, but they are also awkward for fast and progressive transmission in bandwidth-limited applications. Wavelets tackle such issues, the multiresolution structure (Fig. 1) making the progressive processing easier.

A problem for applying wavelets on meshes is the irregular sampling (unlike still images or videos). Despite the development of wavelets for irregular meshes [3,4], a popular solution is to remesh the input mesh semi-regularly (for instance with [5–7]) before applying wavelets. The principle is to resample the surface geometry while providing a subdivision connectivity. The output is called a semi-regular mesh, and wavelet filtering is finally more efficient.

1.1. Related work

Lounsbery et al. are considered as pioneers in the development of wavelets for surface meshes of arbitrary topological type [8]. They proposed a technique to construct wavelets from any local, stationary, continuous, uniformly convergent subdivision schemes such as Catmull–Clark [9], Loop [10], or Butterfly [11]. The subdivision scheme represents the synthesis filter, and the analysis filter is derived from it. Two filters are finally applied on the input mesh during analysis providing, respectively, a mesh of low resolution (low-pass filtering), and a set of wavelet coefficients (high-pass filtering).

Inspired by the work of Lounsbery et al., and by the work of Donoho concerning interpolating wavelet transforms [12], Schröder and Sweldens presented how building wavelets for scalar functions specifically defined on a sphere [13]. They are not the first constructing wavelets on the sphere. The pioneers are Dahlke et al. [14], who used a tensor product basis where one factor is an exponential spline. A continuous transform and its semi-discretization have been also proposed by Freeden and Windheuser [15]. Nevertheless, the work of Schröder and Sweldens in [13] is

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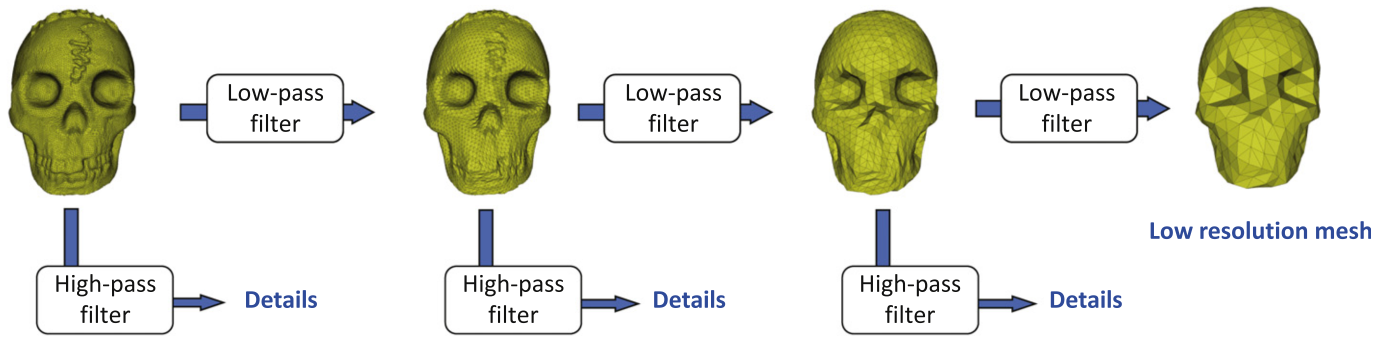

Fig. 1. Overview of a wavelet decomposition.

remarkable because it is the first showing how the *lifting scheme* [16] is particularly relevant to construct biorthogonal wavelets with nice properties, and how the resulting wavelet filters are easy to implement (local vertex-manipulating filters). Moreover, this technique is parameterization independent. Kovacevic and Sweldens then generalized the concept of wavelets for any kind of meshes [17]. They showed that the lifting scheme allows to construct filter banks and wavelets for any lattice, any dimension, and any number of primal/dual vanishing moments. They also showed that only two lifting steps are needed (predict and update), but one condition is that the associated scaling functions are interpolating.

Until this work, most of wavelet transforms for semi-regular meshes were based on interpolating subdivision schemes, in particular on the Butterfly scheme. However, a Loop-based wavelet transform was proposed in 2000 by Khodakovsky et al. [18]. The approximating Loop subdivision scheme is used during synthesis as low pass reconstruction filter, whereas the associated high-pass filter is derived from it by applying a quadrature mirror construction. The drawback of this approach arises during wavelet analysis, because filters cannot be directly applied. Contrary to wavelet transforms based on lifting scheme, the wavelet coefficients and the low resolution mesh are obtained by solving sparse linear systems depending on the two low- and high-pass reconstruction filters. In 2004, Bertram overcame this problem by proposing a biorthogonal Loop-based wavelet construction based on the lifting scheme [19]. This is also the case of Li et al. who proposed in parallel a reversible (but unlifted) Loop-based wavelet transform [20]. Finally, in 2008, Charina and Stöckler proposed to tackle this drawback by using tight wavelet frames [21], which leads to the use of the same scheme during reconstruction and decomposition.

Compression allows compact storage and/or fast transmission in bandwidth-limited applications of massive meshes, and many techniques have been already proposed [22]. To our knowledge, wavelet-based coders that take semi-regular meshes as input are the most efficient, because of their piecewise sampling regularity allowing efficient wavelet decomposition. We briefly present the main works in this domain.

The first wavelet-based coder (often called PGC) for semi-regular meshes was proposed by Khodakovsky et al. [18]. This coder is based on multi-scale quadtree structures and supports quality scalability. The authors propose a Loop-based wavelet transform (presented in previous section), but any wavelet transform could be used. A zerotree coder followed by an entropy encoding are applied in parallel on each component (tangential and normal) of the wavelet coefficients computed in a local frame. This coder has been also proposed for *normal meshes* [23]. The only difference is the choice of the wavelet transform. The authors use the *unlifted* Butterfly-based wavelet transform (*i.e.*, without update step), optimal for this kind of meshes.

Then, several allocation techniques [24–27] were proposed for improving the coding performances of the wavelet coders. The principle is to use a bit allocation process during the quantization step in order to analytically optimize the rate–distortion tradeoff, in other words, reach the maximal quality for a minimal file size (or *vice versa*).

Recently, a coder providing both resolution and quality scalability was proposed by Denis et al. [28]. This coder exploits the intraband or composite statistical dependencies between the wavelet coefficients. By following an information-theoretic analysis of these statistical dependencies, the wavelet subbands are independently encoded using octree-based coding techniques and a context-based entropy coding. This coder provides better results than PGC, and similar results with [24] that is not quality scalable.

1.2. Motivation and contributions

One limitation of wavelets for meshes is that the structure is fixed. For instance, many wavelet coders use the Butterfly-based scheme [13]. From a compression point-of-view, this wavelet is relevant for smooth surfaces because of the interpolating effect of the Butterfly scheme used as predictor, which produces small coefficients. But this scheme is less efficient for other kinds of surfaces, with high frequency variations or salient features, for instance. Finally, a wavelet changing in function of the geometric features of the input mesh could be a relevant tool. When the transforms are lifting-based, this can be finally achieved by *adapting* the predict and update steps [29] to the input mesh.

Therefore we propose an algorithm for optimizing two popular lifting-based wavelet transforms for semi-regular meshes: the Butterfly-based scheme [13], and the Loop-based one [19]. Our motivation is to improve the performances of the state-of-the-art wavelet coders, by adapting the multiresolution analysis tool to the features of the input mesh. The basic idea is to find, for a given semi-regular mesh, the prediction operator that maximizes the sparsity of wavelet coefficients at each level of resolution. Indeed, it is well known in information theory that maximizing the data sparsity improves the coding performances [30].

The idea of adapting the prediction step of the Butterfly-based scheme has been already introduced in [31]. The main contributions of the current paper are:

- More technical details about the optimization algorithm for the Butterfly-based lifting scheme.
- A more robust method for computing the update operator for this scheme. The reason is that the technique proposed in [31] sometimes fails because of a potential null divisor.
- An extension of the optimization algorithm to the Loop-based lifting scheme [19], by taking into account the features of this scheme.

The rest of this paper is organized as follows. Section 2 introduces notions about semi-regular meshes and lifting scheme