Computers & Graphics 35 (2011) 726–732

Contents lists available at ScienceDirect

Computers & Graphics

journal homepage: www.elsevier.com/locate/cag

Short Communication to SMI 2011

Volumetric colon wall unfolding using harmonic differentials

Wei Zeng *, Joseph Marino, Arie Kaufman, Xianfeng David Gu

Computer Science Department, Stony Brook University, Stony Brook, NY 11794, USA

ARTICLE INFO

Article history:
Received 10 December 2010
Received in revised form
14 February 2011
Accepted 7 March 2011
Available online 24 March 2011

Keywords:
Volumetric mapping
Harmonic differential
Virtual colonoscopy
Computer-aided detection

ABSTRACT

Volumetric colon wall unfolding is a novel method for virtual colon analysis and visualization with valuable applications in virtual colonoscopy (VC) and computer-aided detection (CAD) systems. A volumetrically unfolded colon enables doctors to visualize the entire colon structure without occlusions due to haustral folds, and is critical for performing efficient and accurate texture analysis on the volumetric colon wall. Though conventional colon surface flattening has been employed for these uses, volumetric colon unfolding offers the advantages of providing the needed quantities of information with needed accuracy. This work presents an efficient and effective volumetric colon unfolding method based on harmonic differentials. The colon volumes are reconstructed from CT images and are represented as tetrahedral meshes. Three harmonic 1-forms, which are linearly independent everywhere, are computed on the tetrahedral mesh. Through integration of the harmonic 1-forms, the colon volume is mapped periodically to a canonical cuboid. The method presented is automatic, simple, and practical. Experimental results are reported to show the performance of the algorithm on real medical datasets. Though applied here specifically to the colon, the method is general and can be generalized for other volumes.

1. Introduction

Colorectal cancer is the third most incident cancer worldwide [16]. It is recommended that the elderly and at-risk are regularly screened for polyps, the precursors of cancer, located on the colon wall. This has traditionally been accomplished by performing an optical colonoscopy (OC), where an endoscope is inserted into the colon through the rectum. Due to the inherent discomfort in this procedure and its bowl preparation, and the risk associated with this invasive procedure, virtual colonoscopy (VC) systems have been developed [7]. In VC, computed tomographic (CT) images of the patient's abdomen are used to reconstruct a virtual colonic model. Doctors are then able to navigate through this model using a virtual fly-through visualization system and search for polyps. Recent studies have shown such systems to be as effective as or better than traditional OC methods [10].

VC systems, however, share some inherent disadvantages with OC. Inspection of the entire colonic wall can be time consuming due to the length of the colon. Worse, however, is that due to the haustral folds present on the colon walls, many areas are hidden in the standard view, and any polyps in these areas are therefore missed, leading to incomplete examinations. One solution to overcome this limitation is to map the entire surface of the colon to a plane, such that there are no occlusions. Conformal flattening is an advantageous method for this since the resulting mapping is angle preserving and the global distortion is minimized [8]. The property of being angle preserving is especially important, as this means that shapes will be preserved; thus the shape of a polyp will still be readily identifiable to a user (doctor) trained in identifying polyps in 3D. The flattened 2D mesh can be rendered using direct volume rendering with GPU acceleration. In addition to visualization, conformal flattening of the colon surface has been applied for computer-aided detection (CAD) of colonic polyps using electronic biopsy [9] and for registration of supine and prone colon surfaces [31].

The anatomy of the colon wall, however, is intrinsically volumetric. Recent work has shown that it is possible to detect and extract the outer colon wall from the CT images [24]. Utilizing this volumetric data, we propose a method for volumetric colon unfolding, which offers more complete and accurate information. For visualization, rendering the volumetric colon wall can offer better results due to the automatic presence of the volume rendering view vector. That is, given the inner and outer colon surfaces flattened to two parallel planes (i.e., the front and back faces of the cuboid), the view vector can be immediately defined for any point on the surface. Using only a single surface, this view vector must be calculated in some manner (e.g., from the nearest point on the centerline, using the surface normal, etc.), which can lead to slightly inaccurate results. For virtual biopsy in VC and CAD, using the volumetric colon wall in conjunction with volume rendering integration ensures not only that the view ray is correct, but also ensures that the accumulation of the discrete

* Corresponding author.
E-mail address: zengwei@cs.sunysb.edu (W. Zeng).

0097-8493/$ - see front matter Published by Elsevier Ltd.
doi:10.1016/j.cag.2011.03.008

volume rendering integral is finished when the colon outer wall is reached. For supine-prone registration, the volumetric colon presents additional information that can be used to arrive at a more accurate registration result.

Our method is based on three harmonic 1-forms, which can be treated as three vector fields, such that both the divergence and circulation of them are zero. The volume is unfolded to a canonical cuboid by integrating over these three harmonic 1-forms. The proposed method has the following *advantages*:

1. *Rigorous and theoretically solid*: The algorithm is based on Hodge theory.
2. *Automatic*: It also allows for human intervention.
3. *Efficient*: It is to solve large sparse linear systems.
4. *Shape preserving as much as possible*: The global distortion from colon volume to parametric cuboid, measured by harmonic energy, is minimized.
5. *General*: It can be generalized to deal with the volumes with more complicated topologies.

Combined with the direct volume rendering, the 2D colon image provides an efficient way to augment VC systems. The unfolding of a 3D volumetric colon is a fundamental step in the implementation of further virtual colon processing algorithms, such as enhanced CAD and registration techniques, as well as new visualization possibilities.

2. Related work

Visualization methods have been proposed to assist the medical doctor in ensuring a complete inspection during VC, such as the unfolded cube projection method [25]. A popular method for showing the entire colon at once is that of virtual dissection. Early work focused on the straightforward method of using cross sections through the colon for each point by straightening the centerline [28]. However, there may be overlap at neighboring cross sections, causing a polyp to appear twice or not appear at all in the flattened image. An iterative method was applied to this, using two consecutive cross sections at a time, to correct for these overlaps [27]. Electrical field lines generated by a locally charged path was also used to generate curved cross sections instead of planar sections [26]. However, it still cannot guarantee that the curved cross sections do not intersect each other.

Paik et al. [14] used cartographic projections to project the whole solid angle of the camera. This causes distortions in shape. Bartrolí et al. [2] proposed to move a camera along the central path of the colon and map the local regions to different 2D frames through the cylinder ray tracing. This avoids the appearance of double polyps since intersections can only appear between different frames. However, it does not provide a complete overview of the colon. They presented a further two step technique to deal with the double appearance of polyps and nonuniform sampling problems [1]. However, it is important to this method that the central path is smooth and has as many linear segments as possible.

Conformal mapping has also been used to flatten the colon surface to a plane. Haker et al. [6] have proposed a method based on the discretization of the Laplace–Beltrami operator to flatten the colon surface onto the plane in a manner which preserves angles. The flattened colon surface is colored according to its mean curvature. However, the color-coded mean curvature is not efficient for polyp identification, and it requires a highly accurate and smooth surface mesh to achieve a good mean-curvature calculation. Hong et al. proposed a method based on harmonic 1-forms [8], which flattens the colon surface to a rectangle. More recently, Qiu et al. proposed a method for conformal flattening based on discrete Ricci flow [17] to deform the Riemannian metric using the heat diffusion equation of curvature flow with a target Gaussian curvature of zero everywhere.

Many colon CAD systems use the geometry of the colon surface to perform shape analysis for areas that have the appearance of a polyp (bulbous convex structures on colon walls which protrude into the colon lumen). These methods to characterize the curvature measures of the colon wall in CAD systems include shape index and curvedness [30,13], shape description based on global curvature [29], the intersection of normals [15], and sphere fitting [21,11]. CAD of the colonic polyps has also been implemented using the conformally flattened colon surface rendered by direct volume rendering with a translucent transfer function [9].

In recent years, the discrete one-form has been applied to many applications in computer graphics and visualization, such as surface parameterization [5,3], quad mesh design [22], vector field design [23,12], and so on. We refer readers to [4] for a more thorough list of applications and references.

3. Theoretic background

In practice, all the shapes are approximated by tetrahedral meshes, namely, simplicial complexes. In the following discussion, all of the concepts are defined on meshes directly. More details can be found in [4].

Suppose M is a simplicial complex. We use $[v_0,v_1,\ldots,v_n]$ to represent an *n-simplex*. For example, vertex, edge, face, and tetrahedron are simplices.

3.1. Closed and exact forms

A *k-chain* γ_k is a linear combination of all k-simplices in M, $\gamma_k=\sum_i c_i\sigma_i^k$. All k-chains form a linear space, called the *k-dimensional chain space* C_k. The *k-dimensional boundary operator* ∂_k: $C_k \to C_{k-1}$ is a linear operator, defined as taking the boundary of a k-chain,

$$\partial_k\left(\sum_i c_i\sigma_i^k\right)=\sum_i c_i\partial_k(\sigma_i^k),$$

where σ_i^k goes through all k-simplices in M. On each simplex,

$$\partial_k[v_0,v_1,\ldots,v_k]=\sum_{i=0}^{i}(-1)^i[v_0,\ldots,\tilde{v}_i,\ldots,v_k],$$

where $[v_0,\ldots,\tilde{v}_i,\ldots,v_k]$ represents the $(k-1)$-simplex with vertices from v_0 to v_k except v_i.

A *k-form* is a linear function defined on the chain space, $\omega_k : C_k \to R$,

$$\omega_k(\gamma_k)=\omega_k\left(\sum_i c_i\sigma_i^k\right)=\sum_i c_i\omega_k(\sigma_i^k).$$

Sometimes, the action of ω_k on γ_k is also denoted as $\langle\omega_k,\gamma_k\rangle$. All k-forms form a linear space, the so-called *k-dimensional co-chain space* C^k. The *k-dimensional co-boundary operator* $d_k : C^k \to C^{k+1}$ is a linear operator, defined as the dual operator of the boundary operator ∂_{k+1},

$$\langle d_k\omega_k,\gamma_{k+1}\rangle=\langle\omega_k,\partial_{k+1}\gamma_{k+1}\rangle. \quad (1)$$

The above equation is called the *Stokes formula*.

Suppose ω is a k-form, then ω is an *exact form*, if there exists a $(k-1)$-form τ, such that $\omega=d_{k-1}\tau$; ω is a *closed form*, if $d_k\omega$ is 0. It can be verified easily that all exact forms are closed. Let ω_1 and ω_2 be closed k-forms; if they differ by an exact k-form, $\omega_1-\omega_2=d\tau$, then they are *cohomologous*. All the cohomologous