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Medley of spirals from cyclic cellular automata

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ABSTRACT

Cyclic cellular automata on the integer planar lattice are known to typically evolve through distinct phases ending with minimal periodic terminal states that usually appear as intertwined spirals. Here we explore the diversity of spirals that arise from nonstandard neighborhoods on the integer lattice and from looking at the automata on quasi-crystalline arrangements of cells. We see that phase transitions and development of spirals are almost ubiquitous yet the particular form of the spirals is very dependent upon the particulars of the underlying neighborhoods; in fact the spiral forms echo the neighborhoods. The quasi-crystalline illustrations provide much more subtle echoes in the spiral forms that show artifacts from the non-periodic local symmetry that occurs.

1. Introduction

A cellular automaton is a collection of cells that in each time step are in a "state"; usually the set of allowed states for each cell is finite. The cells are affected by a specific local neighborhood and a local rule such that the cells evolve from one time step to the next according to the rule. Cellular automata are interesting because of their simplicity and complex behavior [1–3]. The most famous automaton is Conway's Game of Life which was popularized by Martin Gardner's Scientific American Columns [4,5]. That automaton runs on the integer lattice with nearest eight neighbors and two possible states that are updated according to a simple rule. It is intriguing because periodic structures occur, as do moving configurations and generators. In fact, it is known to be capable of universal computation [6].

Cyclic cellular automata were introduced by Griffeath and co-workers[7] and described by Dewdney [8] in Scientific American in 1989, where they were called cyclic-state automata. We will refer to them as cyclic cellular automata (CCA). When CCA are applied to a random initial configuration, they typically evolve through distinct phases that have different appearances. The end result is visually dramatic, periodic spirals that self-organize. Spiral formation arises in physical situations [2,3,9,10] and in other models [1–3,7,10–13]. While not every question about these automata can be answered, it is possible to see why organized structures should develop, as explained in [7,8,10], and will be described below. Being able to explain this rich behavior is an unusual and wonderful feature of these automata. Generalizations of CCA to wider neighborhoods and thresholds has been studied as well [7,10,12].

A cyclic cellular automaton is defined as an automaton where each cell takes one of N states $0,1,2,\ldots,N-1$ and a cell in state i changes to state $i+1 \bmod N$ at the next time step if it has a neighbor that is in state $i+1 \bmod N$, otherwise it remains in state i at the next time step. The most classic CCA are applied on the 2-dimensional integer lattice with von Neuman neighborhoods (nearest four NWES neighbors). However, this rule can be applied to any configuration of cells and any definition of neighborhood in any dimension. In fact, it can be applied to any graph.

In this investigation we explore CCA on a rich variety of planar graphs. These include several types of nearest-neighbor neighborhoods on the integer lattice and quasi-crystalline graphs. There are many kinds of neighborhoods that can be formed even in this case where only nearest neighbors are considered and the resulting spirals include classical diamonds, squares, and variants that echo the neighborhood in an inverted sense. The quasi-crystalline arrangements of cells lead to many-sided spirals that are echoes of more subtle local symmetry appearing in those configurations.

2. Debris, defects and demons

Before giving definitions, we give a classic illustration. Fig. 1 shows CCA using von Neuman neighborhoods on a 500 by 500 array of cells. Periodic boundary conditions are used so that cells on the right edge have neighbors from the left edge and analogous comments apply to the left, top and bottom edges. Four different time steps are shown in Fig. 1. The automata were implemented in J [14] using the automata templates from [15]. The cyclic cellular automata uses $N = 14$ states and the initial states for each cell are chosen randomly and uniformly from those states. The states are shown cyclically with hue, running from red through

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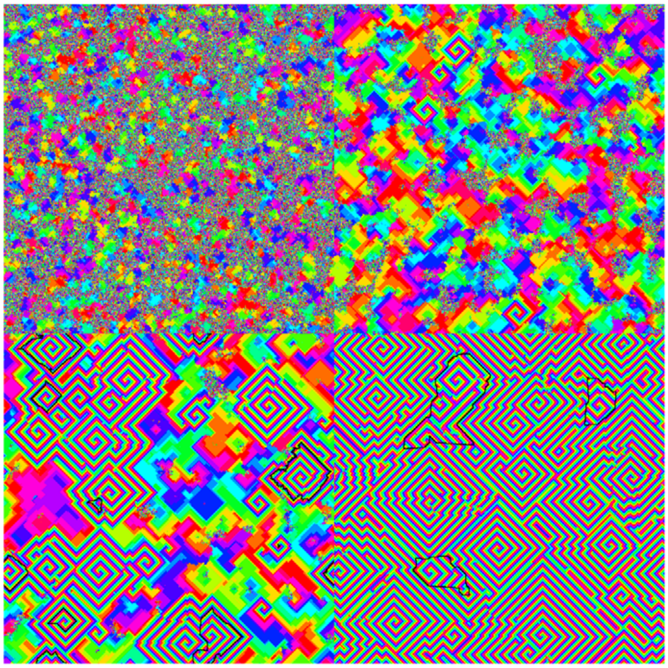

Fig. 1. Classic CCA with $N=14$ showing droplet formation, spiral formation and demon domination.

intermediates to green, and then blue and magenta as the states run from 0 to $N-1$. The upper left portion of the figure occurs after 75 time steps and one can observe rough "debris" regions that have not substantially evolved and "droplets" of color, with some waves of changes moving across them. The upper right shows the result after 150 time steps and we see the droplets cover a majority of the region, but some spiral "defects" have begun to evolve. After 225 time steps, shown on the lower left, we see many spirals formed and some of them have grown quite large. Here the shortest possible nontrivial periodicity that can occur is 14 and the black pixels mark the edges of regions that have repeated with that optimal periodicity. The spirals (defects) that are surrounded by black pixels have optimal period, and are called "demons". The bottom right of the figure shows the situation after 975 time steps. At this point the demons have overtaken most of the other spirals and the black pixels surround small regions that are defects with periods higher than 16, which will soon also be overtaken by the demons. Many movies showing the evolution of the CCA described in this paper may be found at [16]. Watching the phase transitions evolve reinforces the dynamic nature of these processes.

Following [7,8] we describe these phases more carefully. The bond between two neighbors is *open* if the difference between their states is $\{-1,0,1\} \bmod N$. Otherwise the bond is *closed*. Note that once a bond is open it must remain open for all future time. A site is considered *debris* at a given time if it has no open bonds with its neighbors. The connected components of the non-debris sites are called *droplets*. A loop within a droplet using open bonds could possibly include only bonds $\{-1,0,1\} \bmod N$. If one takes a running sum (not $\bmod N$) of the differences $\bmod N$ (so each difference is from $\{-1,0,1\}$) along such a loop, then the total must be zero $\bmod N$ since each cell value appears once in a positive and a negative sense. Thus, the sum of the differences must be a multiple of N. If the multiple is not zero, then the loop is part of a *defect*. That property of the loop will be preserved as the automaton evolves, and each cell in a defect must eventually continue cycling through the states $0,1,2,\ldots,N-1,0,\ldots$ forevermore; although it may take more than one time step between each change of state. Eventually every cell neighboring such a loop must cycle. Then their neighbors must cycle. Eventually every cell in such a droplet must cycle, and thus the droplet must grow since every neighbor of the droplet will eventually join the droplet since the neighbor in the droplet will eventually cycle through all values. Of course the speed of such cycling need not be constant until it is part of a demon. If a defect loop runs through all states as efficiently as possible on the lattice (minimal period having no unnecessary 0 bonds) then it is a *demon*. Given random initial states for all the cells on an infinite lattice, demons are expected to occur somewhere with probability one. Thus, demon domination is the expected long-term state.

Fig. 2 shows zooms into two of the spirals from the end state of the experiment shown in Fig. 1. The closed bonds are demarcated by black lines. On the left is a demon since if one marches around the "L"-shaped closed bonds, one moves through the 14 states using 14 cells crossing 14 bonds; all those bonds correspond to one-level change in state, all the same in a positive or negative sense. That property persists for all future times and thus that spiral is a demon. The spiral on the right is a defect, but not a demon. Any short path around the closed bonds using left–right and up–down bonds requires more than 14 cells and visits yellow and violet cells twice. Such a loop must persist but changes can (and will, with probability one) occur to the closed bonds so that this region becomes part of a more efficient, minimal period, defect that is a demon.

However, the arrangement of cells may not allow for a demon loop of length N. For example, with von Neuman neighborhoods, it is not possible for a loop to return to itself using an odd number of bonds. Thus, if $N=15$ then a demon loop must contain at least one bond where there is no change in state. Fig. 3 shows this situation where a loop around the closed bonds visits light green twice. Also note debris, droplets, defects and demons are all apparent after 255 time steps.

3. Moore neighborhoods

We now turn to discussing other neighborhood patterns using sub-neighborhoods of the 3 by 3 neighborhood surrounding the cell. We number a 3 by 3 neighborhood so that the center has number 4 as shown in Fig. 4. Letting P denote the neighborhood pattern, we write $P=1357$ for the von Neuman neighborhoods used in the previous section. The complete Moore neighborhood thus has pattern given by $P=01235678$. The behavior of the CCA with this sense of neighbor is for the most part qualitatively similar to what we saw in Fig. 1, but some distinctions should be made. Fig. 5 shows the $N=20$ state evolution at times 63, 105, 147 and 231, respectively. Notice the droplets form spirals develop, Notice the droplets form, spirals develop, dominating and

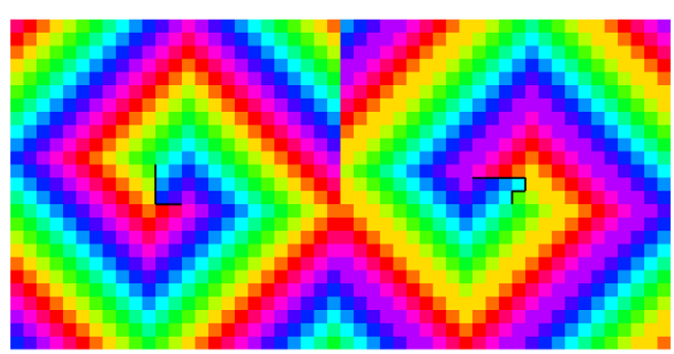

Fig. 2. CCA with $N=14$. A demon and a non-demon defect with inactive bonds shown in black.