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Impossible fractals

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Abstract

Impossible objects are a type of optical illusion involving ambiguous visual descriptions of figures that cannot physically exist. It is shown by way of example that such objects can be further developed using standard fractal techniques to create new, more complex designs that retain the perceptual illusion, sometimes allowing additional illusions to emerge from the process. The balanced Pythagorean tree is used to efficiently render impossible fractals that display the perceptual effect across decreasing levels of scale.

Keywords: Impossible object; Optical illusion; Fractal; Art

1. Introduction

In the field of psychology, there is a long tradition of optical illusions that exploit quirks in visual perception to create misleading figures with ambiguous or contradictory perceptual interpretations. One such type of illusion is the *impossible object*, which is a shape that cannot physically exist despite having an apparently valid visual description. Martin Gardner [1] describes such objects as *undecidable figures*.

A defining characteristic of impossible objects is that each part makes sense but the whole does not; local geometry is satisfied but the figure's global geometry is ambiguous or contradictory, and the viewer must constantly revise their understanding of the figure as their eye travels over it. As Penrose and Penrose [2] put it, each individual part is acceptable but the connections between parts are false. Many examples of impossible objects can be found in Bruno Ernst's *The Eye Beguiled: Optical Illusions* [3] which provided the inspiration for most of the constructions in this paper.

In the field of mathematics, there is a long tradition of objects that display fractal geometry, even though the precise definition of self-similarity that underpins them and their classification as a related group is relatively new. Classical fractals typically involve simple transformations recursively applied to simple shapes to produce more complex shapes. *Chaos and Fractals: New Frontiers of Science* by Pietgen et al. [4] provides a comprehensive overview of fractals, their construction and basic properties.

When drawing impossible objects, artists tend to choose shapes that are as simple as possible in order to emphasise the illusion. This paper investigates whether fractal techniques can be applied to impossible objects to produce new, more complex designs which retain the perceptual effect. The following sections examine some of the more common types of impossible objects, and their development by standard fractal techniques.

2. The tri-bar

The tri-bar (Fig. 1, left) is described by Ernst [3] as the simplest yet most fascinating of all impossible objects, and is one of the most widely recognised. The illusion is created by the ambiguous use of parallel lines drawn in different perspectives, so that the figure appears to perpetually turn out of the page when traversed in a clockwise direction. The two corners at the end of each bar are interpreted as lying perpendicular to each other, which Ernst points out would give a total internal angle of 360° and hence defy a fundamental property of triangles; this figure cannot be physically constructed as a closed shape with perpendicular corners and straight arms.

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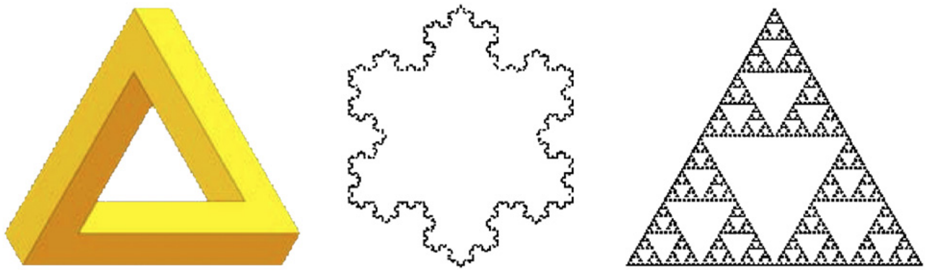

Fig. 1. The tri bar, the Koch snowflake and the Sierpinski gasket.

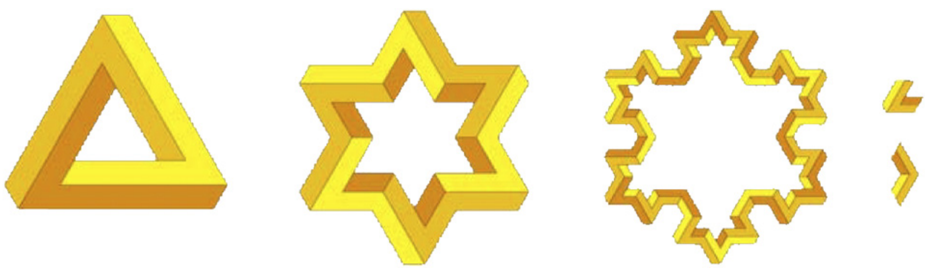

Fig. 2. Two iterations of an impossible snowflake (with acute and obtuse generators shown).

Fig. 3. An alternative snowflake design that emphasises the perceptual effect (with generator shown).

The tri-bar was invented in 1934 by Oscar Reutersvärd, a Swedish graphic artist who went on to become the world's greatest exponent of impossible figures, producing several thousand until his death in 2002. The tri-bar is often called the Penrose Triangle after mathematician Roger Penrose, who independently rediscovered it and popularised it in the 1958 article "Impossible Objects: A Special Type of Visual Illusion" co-written with his father [2]. It is also known as the Escher Triangle as Dutch graphic artist Escher [5] embraced the principles it represented and included its design in many of his works, most famously the perpetual stream of his 1961 lithograph "Waterfall".

We call this figure the tri-bar in keeping with Ernst's terminology [3], which may be extended to multibar figures with more than three sides. Multibars are generally drawn on an isometric grid with the following design rules in mind:

(1) local geometry and shading should be consistent;
(2) adjacent regions should not share the same colour; and
(3) he least number of colours should be used (three colours will generally suffice, although four are required in some cases).

3. Triangular fractals

Fig. 1(middle and right) shows two well-known fractal developments of the triangle, the Koch snowflake and the Sierpinski gasket. The snowflake modifies the triangle's perimeter shape while the gasket recursively subdivides its interior.

Fig. 2 illustrates a development of the tri-bar as an impossible snowflake. The first iteration can be constructed entirely from a single subshape, the acute generator (top right), repeated six times in a cycle with appropriate colouring. Further iterations require a combination of acute and obtuse generators.

Fig. 3 shows an alternative snowflake development that retains parent triangles from previous generations and uses them as a framework upon which subsequent triangular struts are added. Although this is not a traditional snowflake and the final design is busier than the previous figure, this approach only requires a single generator (right) and the struts enhance the ambiguity of perspective to give a stronger effect.

In both cases, the thickness of all bars in the figure are uniformly reduced with each iteration to retain the shape's