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Deformation similarity measurement in quasi-conformal shape space ☆

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ABSTRACT

This paper presents a novel approach based on the shape space concept to classify deformations of 3D models. A new quasi-conformal metric is introduced which measures the curvature changes at each vertex of each pose during the deformation. The shapes with similar deformation patterns follow a similar deformation curve in shape space. Energy functional of the deformation curve is minimized to calculate the geodesic curve connecting two shapes on the shape space manifold. The geodesic distance illustrates the similarity between two shapes, which is used to compute the similarity between the deformations. We applied our method to classify the left ventricle deformations of myopathic and control subjects, and the sensitivity and specificity of our method were 88.8% and 85.7%, which are higher than other methods based on the left ventricle cavity, which shows our method can quantify the similarity and disparity of the left ventricle motion well.

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1. Introduction

Recently, the scanning technologies, such as Magnetic Resonance Imaging, and Positron Emission Tomography, have been advancing rapidly, many of which can be used to capture motions of dynamic objects, such as cardiac motion, face expressions, gestures. A new grand challenge arises in analyzing this type of temporal motion data, especially when there is a necessity to visualize and compare the deformation behavior across subjects. Appropriate deformable shape analysis techniques are of utmost important for this type of time-varying shape comparison and classification. From a physical point of view, the behavior of shapes is governed by physical principles. Therefore, physically based approaches, such as deformable models, are employed to approximate the object deformation by minimizing the summation of the deformation energy under the constraint of smoothness of the model in the Lagrangian setting [1]. Then, the analysis of the derived deformable model can be achieved through finite element analysis.

In contrast, from a geometric point of view, in order to support efficient shape characterization, a higher level of shape abstraction and information reduction is necessary. In machine vision techniques, a shape descriptor extracts a geometric feature from the shape, which is either global feature such as boundary or volume of the shape [2], convex-hull packing [3], or a local feature such as mean or Gaussian curvature and edge length [4]. Then, an energy functional based on the shape descriptor is minimized to classify the shapes. The modern geometry introduces shape space [5,6], where coordinates of points in this space represent some generalized properties related to various geometrical properties. In other words, a shape space is established such that each surface group relates to the same point in shape space. This inspires us to innovate the geometry-based approaches for dynamic shape analysis and shape deformation classification.

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In shape space, a deformation sequence is shown by a curve. The geodesic distance between two points yields the similarity between two shapes according to the property which shape space preserves. Lipman et al. [7] estimate the deformation between two isometric manifolds by minimizing a geometric expressing functional. Huang et al. [8] suggest a constrained energy function based on the gradient domain techniques for deformation estimation. Zhou et al. [9] introduce the Volumetric Graph Laplacian technique to minimize a quadratic energy function preserving the volumetric details during the 3D deformation. Huang et al. [10] present a geometrical potential function with constant stiffness matrix to speed up the deformation approximation. Funck et al. [11] minimize an energy function based on the divergence free vector field to get a smooth, volume preserving deformation. Xu et al. [12] deal with the deformation as gradient field interpolation, and propose a novel shape interpolation approach based on the Poisson equation. However, all these methods directly estimate one diffeomorphism between two shapes by minimizing an energy functional under some constraints, which increases the inconsistency and instability of these methods, especially, in high-dimensional shape spaces.

To address this problem, Kilian et al. [13] present a geometric structure for isometric deformations, which encodes all smooth groups of diffeomorphisms mapping two objects together. The desired characteristics to which shape space is invariant can be induced by choosing an appropriate geometric structure. During the deformation, the Gaussian curvature at each point on the manifold may change according to the deformation characteristics. Based on the spectral geometry, the eigenvalues of the Laplace–Beltrami operator can serve as numerical fingerprints of 2D or 3D manifolds [14], which can also be used to build shape space invariant to isometric deformations. Completely invariant to the conformal transformations, a conformal structure based on the period matrix and related algorithms to calculate the period matrix for manifolds with arbitrary topologies are introduced in [15,16]. Wang et al. [17] further provide a 3D matching framework based on the least squares conformal maps. To induce conformal mappings in 2D and quasi-conformal mappings in 3D, the Green Coordinates [18] are used in the cage-based space deformation estimation. Hurdal et al. [19] and Haker et al. [20] computed quasi-conformal and conformal maps of the cerebral cortex, respectively.

Continuous Ricci flow [21] conformally deforms a Riemannian metric on a smooth surface such that the Gaussian curvature evolves like a heat diffusion process. Eventually, the Gaussian curvature becomes constant and the limiting Riemannian metric is conformal to the original one. In discrete case, the circle packing metric [22] determines the discrete Gaussian curvature, and the discrete Ricci flow [23,24] conformally deforms the circle packing metrics with respect to the Gaussian curvatures. In [25], the geodesic lengths of homotopy classes, measured by Hyperbolic Uniformization metric, is used to determine the coordinates of each conformal class in the Teichmüller shape space to classify the shapes with negative Euler number. In fact, surfaces with the same conformal class share the same Uniformization metric and can be used to classify the surfaces [26]. Some Researchers use the surface Ricci flow method to compute the Teichmüller shape descriptor and analyze abnormalities in brain cortical morphometry, e.g. in patients with Alzheimers disease [27].

In the real world, the deformation characteristic is determined by the elasticity of the material structure undergoing the deformation. Zeng et al. [28] conducted a series of experiments to verify whether natural surface deformations are conformal. They reported that isometric or conformal mappings are rare in the real world, and diffeomorphisms between surfaces which are induced by natural deformations are quasi-conformal mappings. The diffeomorphisms are complex value functions which have one-to-one correspondence to the space of Beltrami coefficients. This one-to-one correspondence can be used for shape analysis between registered surfaces [29,30], as the space of Beltrami coefficients is a simpler functional space that captures the essential features of surface maps. Lui et al. [31] propose a simple representation of surface diffeomorphisms using Beltrami coefficients to facilitate the optimization of surface registrations. Their method reconstructs a unique surface map using Beltrami coefficients and Beltrami Holomorphic Flow, and converts the variational problems of diffeomorphisms to that of Beltrami coefficients. Also, Zeng et al. [28] propose an algorithm to register surfaces with large deformations using quasi-conformal curvature flow method. Their method can circumvent the local minima with appropriate normalization conditions. In the study of anatomical configurations, one should consider the anatomical differences across subjects in order to increase the statistical analysis accuracy within anatomically defined regions of interest. The Large Deformation Diffeomorphic Metric Mapping (LDDMM) method [32] provides diffeomorphisms to map anatomical configurations to extrinsic atlas coordinates. Therefore, the anatomical variations between different configurations are removed and the group-wise analysis can be carried out across different anatomic regions, e.g. enhancement of functional data of medial temporal lobe (MTL), and detection of shape abnormality of the left hippocampus in patients with dementia of the Alzheimer type [33,34].

In this study, we employ the structure proposed in [13], in which the geometrical properties of the deformations can be enforced by choosing a suitable metric in shape space. We present a novel quasi-conformal metric to classify and comparatively analyze the deformations which are following the transformations which are not rigid, iso-metric or conformal. This metric is a symmetric form and utilizes the local changes of curvature to measure the similarity between deformations. One challenge in development of the quasi-conformal geometry is to compute effective numerical quasi-conformal mappings. Although there are some numerical quasi-conformal mapping techniques in the literature solving differential equations with finite elements, they deal with simple domains and cannot be applied on general regions. Zeng et al. [35] propose an algorithm to numerically compute the quasi-conformal mapping on general Riemann surfaces of any genus. The framework we utilize in this study numerically minimizes an energy function to find the geodesic curve connecting